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The effects of churn on the growth of subscription services: Adopters, users, money

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ABSTRACT

Subscription-based service model, where profitability arises from sustained service relationships with consumers, has emerged as the dominant business paradigm across various industries. A notable characteristic of the growth in service markets is the indirect relationship between adoption and monetization. While adoption marks the initial stage of user engagement, monetization occurs gradually as users integrate the service into their routines over time. Consequently, the focus has shifted away from emphasizing ***adoption*** rates to prioritizing the total number of ***users.*** The difference between adopters and users is due to the fact that not all users integrate the service to their routine and some (or many) of them ***churn*** away from the service. The growth of adopters and users, and the ensuing monetary growth, are highly affected by churn, hence the critical issue we investigate in this paper is the valence and size effect of churn on adopters, users and revenues of the firm.

We build on the service modeling approach of Libai, Muller, and Peres (2009) to first explore the impact of churn on dynamics of growth for new subscription services. We explain how churn affects key interest topics, such as the size and time to peak for adopters and users, the market potential of those who have not adopted yet, adopter categories, and conversion of users to money. We hope this work can motivate further explorations of this critical area for new product research in marketing.

1. Introduction

The origins of the Software as a Service (SaaS) revolution are often attributed to Marc Benioff, the founder of Salesforce. In the late 1990s, Benioff identified a significant limitation of the prevailing software licensing model: Firms were required to pay substantial upfront licensing fees, even as rapid technological advancements rendered software applications quickly obsolete. This inefficiency resulted in slow adoption and limited diffusion of enterprise software solutions, which could have proliferated more rapidly under a subscription-based pricing model (Bhattacharya and Bhattacharya, 2021; Miller, 2018).

The subscription-based service model – where profitability arises from sustained service relationships with consumers – has emerged as the dominant business paradigm across various industries (Tzuo & Weisert, 2018; Chen et al., 2018). This shift is not confined to Software alone: Service-oriented strategies now underpin the dominant logic of marketing, even

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in sectors traditionally considered product-based (Rust and Huang, 2014; Vargo and Lusch, 2004). The proliferation of digital technologies has further accelerated this trend, fostering a "servitization" process, where firms increasingly embed services into their offerings (Favoretto et al., 2022; Kowalkowski et al., 2017).

A known example of this transition is the software firm Adobe, which specializes in the creation and publication of content. In 2012 Adobe decided to change its financial model from a perpetual license purchase model to a subscription model (Chatterjee, 2024). With the subscription model, Adobe could release regular updates and new features to its software products. It provided subscribers immediate access to the latest versions without the need to purchase upgrades separately and enabled users to sync and access their work across devices and collaborate with others more effectively. While at the time the industry was quite skeptical of this move, ten years later, it became the default for other software companies. Goods and hardware firms such as Nespresso, Philips, Caterpillar, and Xerox have also moved part of their offerings toward a subscription model (Pathak, 2023). Thus, it has been argued that we may need to move the term SaaS to XaaS – everything as a service (Bertini and Koenigsberg, 2020; IBM, 2024). This change is also reflected in the world of new products and ventures. About half of all venture capital funding goes to companies with a SaaS business model (Ho, 2024), and the new venture business literature largely assumes that new product adoption is followed by usage and churn (Bessemer, 2023; Ries, 2011).

A notable characteristic of the growth in service markets, particularly in terms of financial performance, is the indirect relationship between adoption and monetization. While adoption marks the initial stage of user engagement, monetization occurs gradually as users integrate the service into their routines over time. Consequently, the focus has shifted away from emphasizing ***adoption*** rates to prioritizing the total number of ***users***, which better corresponds to the revenue stream of service-based business models. The difference between adopters and users is due to the fact that not all users integrate the service into their routine, and so some (or many) of them ***churn*** away from the service.

Consider SiriusXM Satellite Radio whose annual churn is 18 %, and the current number of subscribers is 33m.[1] With these figures, the firm loses about 6m subscribers annually. This loss has several consequences, all eventually translate to profitability: SiriusXM has to acquire 6m new subscribers annually just to remain with its current number of subscribers, and any growth has to come with new subscribers above the 6m threshold. Moreover, future growth and eventual market potential will be affected by this loss of 6m subscribers as SiriusXM will not benefit from their word-of-mouth effect, and in effect can suffer from their negative contagion effect. Thus, growth of adopters and users, and the ensuing monetary growth, are highly affected by churn.

Next, we extend the paper by Libai, Muller and Peres (2009), to investigate the valence and size effect of churn on adopters, users and revenues of the firm. It is important to determine our boundaries in this paper: We focus here on the growth of new services rather than mature ones, and the services we consider here are recurring services where individuals use the service each period until they churn. Previous research has considered such services also under the term *subscriptions* (Ben Rhouma and Zaccour 2018; McCarthy, Fader and Hardie 2017) or *contractual services* (Fader and Hardie, 2015). To clarify the type of product we focus on, we will use the term subscriptions here as well. However, we use it in the broader meaning of a recurring consumption service where a monthly fee is not necessarily required (such as in many Freemium models) or where the term "subscriptions" is not always used in practice (such as for various financial services).

Our work using analytical modeling and simulations helps to highlight the effects of churn on key features of new subscription services growth, such as the size and time to peak for adopters and users, the market potential of those who have not adopted yet, adopter categories, and how churn can further affect the transition from usage to revenues and profitability. Given the scope of the new product growth literature and the multitude of ways churn can be integrated into it, the potential for using new service growth models to understand how subscription services grow is large, and we hope this analysis can serve as a step in this direction.

2. Background: Customer churn and the growth of service subscriptions

Since the shape and dynamics of growth of products and services are central to our understanding of new product management (Chandrasekaran and Tellis, 2018), an intriguing question arises: can we apply traditional new product growth concepts in a world where services drive growth? The question is particularly important since, historically, most diffusion of innovation models focused on the first purchase, and even when the growth of service products was analyzed, they were examined mainly via models focused on the first adoption in the market (Peres, Muller, and Mahajan 2010). Yet the eventual aim of marketing managers is to create and accelerate cash flows, not only adoptions (Srivastava et al. 1998). Since durables' monetization happens at adoption, the first purchase growth dynamics can provide a reasonable proxy for the monetary dynamics of a new product. But there is a discrepancy between adoption and monetization for services, so adoption loses much of its relevance in describing the monetary growth process.

One consequence is that the number of users, not adopters, becomes the center of attention and reporting for many subscription firms. Social media entities, such as Facebook and Twitter, streaming entertainment subscriptions, such as Spotify and Netflix, and direct-to-consumer firms, such as Blue Apron and Dollar Shave Club, are examined and evaluated on the pattern of user growth. There is further interest in practice in monetary growth. Period-by-period measures such as Annual

[1] Source: SiriusXM 2024 Annual Report.

Recurring Revenue (ARR) have become essential growth measures for technological markets (Elhafed, 2024; MetricHQ, 2024; van der Kooij, 2015).

Hence, the growth of new subscriptions should be examined via three curves of growth: adopters, users, and money. And the prominent parameter to understand these curves is customer churn (or its complement, retention). Churn will impact the adopter curve because the number of users affects growth due to social influence, such as word of mouth and network effects. It will affect the user curve because the number of users over time depends on the churn rate. And it will affect the amount of money created during each period. It is no wonder that customer churn has become a key measure of interest for consultants, investors, and businesses dealing with new products and ventures (Bessemer, 2023; Eisenmann, 2021; Faster Capital, 2024; Ries, 2011).

However, the academic literature is limited on how customer churn drives the growth curves of subscription. There is a rich literature on how customer retention (churn) affects customer and firm profitability and how it can be predicted, identified, and proactively managed (see Ascarza et al., 2018; Muratcehajic and Loureiro, 2024 for reviews). Yet this literature, which mainly evolved in mature services, does not explore the effect of churn on growth. From the new product growth research side, the shape and dynamics of growth are essential for the ability to predict, launch, value, and manage new products. However, these analyses were primarily done on first-purchase models (Chandrasekaran and Tellis, 2018).

The analysis of product growth in marketing and related disciplines has been done mostly via diffusion models such as the Bass model and its extensions that capture the bell-shaped and the S-shaped cumulative nature of adoption growth, particularly for durables (Meade and Islam, 2006; Peres et al., 2010). Early in this research stream, researchers also considered cases of growth where physical goods were purchased more than once. These included durable replacement, additional items, successive generations of technology where one durable generation replaces another, or trial-repeat models for the case of consumer-packaged goods (see Ratchford et al., 2000). The attention, however, has begun to shift to service growth due to its rising share of the economy, the technological ability to manage customer data, and the desire to better tie customer management to the firm's financial outcomes (Rust and Chung, 2006).

Gupta, Lehmann, and Stuart (2004) provided an early modeling of service growth, combining the lifetime value of each customer acquired with a diffusion model to assess the customer equity of new service firms (see Table 1 for research in this area). However, they did not incorporate the retention rate into the growth process and, therefore, did not consider the effect of retention on acquisition via social influence from existing customers. Libai, Muller, and Peres (2009) were the first to model a Bass-type diffusion growth, where customers can churn after adoption. They identified a closed-form solution to this process and used it to assess customer equity also in a competitive scenario.

The papers that followed in this regard mostly took an aggregate growth modeling approach in the spirit of Libai, Peres, and Muller (2009). They considered, for example, the effect of customer equity on shareholder value (Schulze et al., 2012), acquisition-retention optimization tradeoff (Ben Rhouma and Zaccour, 2018), and marketing mix optimization (Mesak et al., 2022). McCarthy, Fader, and Hardie (2017) focused on using data from subscription firms' financial reports to assess customer equity and could thus build a model that uses rich cohort-level data. McCarthy and Fader (2018) extended this approach to non-contractual relationships.

The marketing literature sees three main ways individuals create value for the firm (Du et al. 2021): *acquisition* of new customers, *development* of existing customers (cross-selling, up-selling, higher markup, and higher purchase frequency), and *retention* of existing customers. Fig. 1 presents the three types of subscription growth curves. One can distinguish between the overall and net curves within each curve. For example, adopters over time are the S-shaped cumulative curve, while net adopters are the new adopters' bell-shaped curve.

To that, we can add a fourth element, *influence*: How customers influence other customers' acquisition, development, and retention. The four elements are the base of what is sometimes labeled in practice as the customer life cycle (Agility, 2022; Saasquatch, 2023) and can help to explore the creation of the three curves.

Adopters Growth: Starting with the adoption curve in Fig. 1, the firm's customer acquisition efforts naturally affect the adoption growth. However, it is also impacted by social influence from previous adopters through word of mouth, observational learning and norms, and network externalities (Peres et al. 2010). Due to the social impact, customer churn will also affect adoption: When people disadopt, the number of previous adopters can influence the number of prospective adopters to go down, which will affect the speed of adoption (Hogan et al., 2003).

Users growth: Fig. 1 presents the change in users over time or *net users*. The latter's growth is based on adoption growth and thus is influenced by the factors that affect adoption growth. However, it is further impacted by customer churn since only some of the adopters stay as users. The number of users at each point in time is a function of the size of previous cohorts and the time that has elapsed since they adopted the subscription. Social influence can also play an indirect role since the churn decisions of individuals are affected by their environment, particularly the churn decisions of others (Landsman and Nitzan, 2020; Moldovan et al., 2017). Therefore, churn affects the user curve in two ways: first, it affects the shape of the adoption curve, and second, it affects the transition from adoption to usage.

Money growth*:* Different approaches can be taken to form the money curve. Managers and investors of subscription firms often focus on the short-term revenue creation by highlighting the Annual (or monthly) Recurring Revenue (ARR, see MetricHQ, 2024). However, the ARR approach does not necessarily consider costs and specifically does not consider customer acquisition cost (CAC), so the full picture of profitability is lacking. In an alternative longer-term approach, the CLV, or the CLV-to-CAC ratio, is the focus (Gupta et al., 2004; Ofek et al., 2022). The tradeoff between a short-term and a long-term approach to monetary growth has been relatively little explored and is yet of much interest (Schulze et al., 2012). Next,

Table 1
Previous modeling of growing service markets.

Article	Focus of paper	Growth model	Attrition type	Main insights
Gupta, Lehmann, and Stuart (2004)	Measuring customer equity of a growing product for customer-based valuation	Technological substitution logistic model – a simplified Bass model (without an external coefficient)	Fixed Lost-for-Good retention; Churn does not affect growth	Customer value provides a good proxy for firm value; Customer equity is highly sensitive to retention rate
Libai, Muller and Peres (2009)	Measuring customer equity of a growing firm that takes retention into account in both non-competitive and competitive cases; Comparing customer equity to market cap	Extended Bass model that includes retention in both category and brand level cases	Customers who churn join the pool of future adopters; Retention affects growth via a social process	Services growth should be modeled taking the effect of retention on growth into account and can serve as the basis for customer equity calculations; Customer equity assessments were generally close to stock market valuations
Schulze, Skiera and Wiesel (2012)	Linking customer and financial metrics to shareholder value via customer equity	Technological substitution logistic model (simplified Bass model)	Fixed Lost-for-Good retention; Churn affects growth	Debt and nonoperating assets impact the influence of customer equity on shareholder value; Support for infinite horizons in customer-based valuation models
McCarthy, Fader and Hardie (2017)	Using data from public financial reports for the valuation of subscription-based business	Time to adoption is modeled as a split-hazard model	Heterogenous dynamic Lost-for-Good retention	Public data can be used for customer-based valuation; There is a need to fully model the dynamics of acquisition and retention when valuing firms
McCarthy and Fader (2018)	Using data from public financial reports for the valuation of non-subscription-based business	Time to adoption is modeled as a mixture of hazard models	Repeat order timing model	The methodology suggested better predicts sales than alternative models; Noncontractual relationship value can be estimated based on public disclosures
Ben Rhouma and Zaccour (2018)	Optimizing customer acquisition and retention to maximize the customer equity of a growing firm	Simplified Libai Muller and Peres model (without contagion effects); The firm affects acquisition and retention	Customers who churn join the pool of future adopters; Retention affects growth	Optimal acquisition and retention investments are constant in the absence of contagion; Changing acquisition spending given contagion effects
Mesak Scott and Bari (2022)	The effects of marketing mix variables on subscription-based growth	Modified Libai, Muller and Peres model that considers advertising and price	As in Libai, Muller, and Peres (2009)	Including marketing mix variables improves fit and predictive ability; Difference between the maturity and growth stages

our aim is to better understand how churn affects the curves, particularly the user and the money curve. To do so, we need a growth model that takes churn into account.

3. Churn in a subscription growth model

Consider the case of the music streaming firm Spotify. Potential users make three-tiered decisions: Adopt the subscription (a discrete decision), decide about the content and engagement level (a continuous decision), and disadopt the subscription (a discrete decision). Spotify offers two options to the potential user: A flat-rate premium subscription or a limited-options, ad-supported service. While in the former option, churn is defined as a user who stopped paying for a service, the latter depends on the definition of the active user. Spotify defines an active user as a consumer who has consumed content in the last thirty days of the period. While the use of 30 days is subjective, churn can be uniquely computed.

To simplify the exposition, we present the case of the premium part of Spotify as follows: Table 2 and Fig. 2 present the number and growth of Spotify's premium active users – those actively using the music streaming service – and the number of new adopters of the premium service. To understand Table 2, it is important to define certain terms that are essential to the subscription growth framework:

- **Adopters** – $a(t)$ – number of cumulative adopters.
- **Net Adopters** – da/dt – change in the number of adopters over time.
- **Active Users** – $x(t)$ – number of users who are actively using the subscription.
- **Net Users** – dx/dt – change in the number of active users over time.

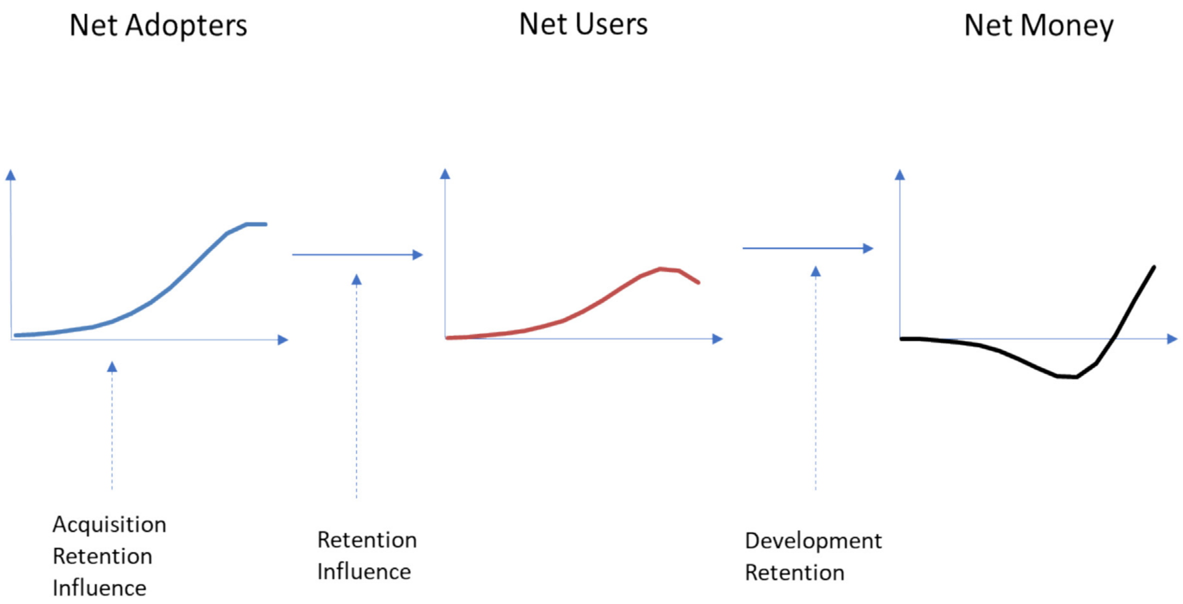

Fig. 1. The subscription life cycle curves.

Table 2
Users and **adopters** of Spotify premium subscription services, worldwide (in millions) *.

	Spotify active users at the end of the year (in millions)	
2024	x_{t+1}	263
2023	x_t	236
Net addition (net users)	$x_{t+1} - x_t$	27
Churn	δx_t ($\delta = 17.9\%$)	42
Actual number of new adopters in 2024 (net adopters)	$x_{t+1} - x_t + \delta x_t$	69

* Source: Spotify Q4 2024 report.

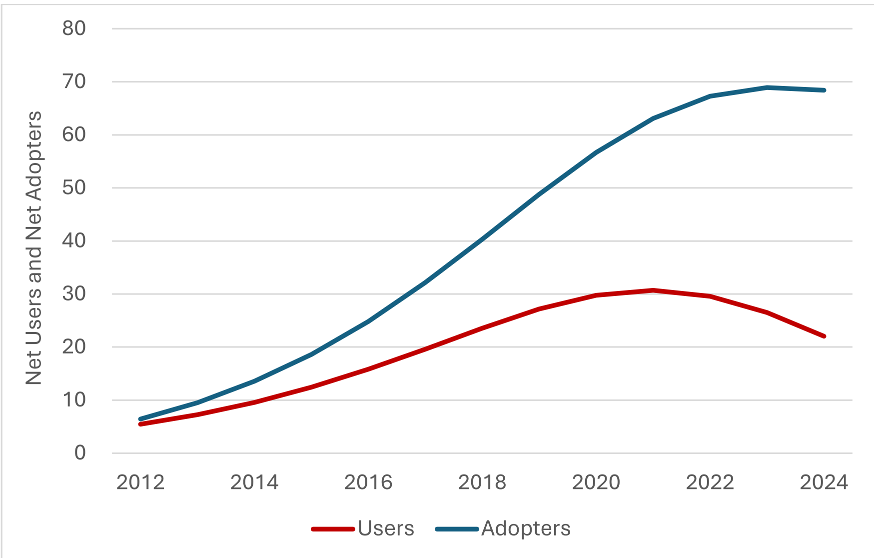

Fig. 2. Growth of Spotify's premium net users and net adopters (in millions) (Source: Spotify's annual reports and Statista).

- **Churn** – δ – percent of active users who decide to leave the subscription.

Note that the net addition to the number of Spotify premium users in 2024 is 27 million. However, during this year, about 42 million users left the subscription service, as the churn rate is about 18 %.[2] Thus, the number of actual new adopters in 2024 is 69 million new users. Given these definitions, the relation between new adopters and new users is given by Equation 1, and note that for Table 2, dx/dt and da/dt could be written as $x_{t+1} - x_t$ and $a_{t+1} - a_t$ in the discrete time version:

$$da/dt = dx/dt + \delta x \tag{1}$$

Fig. 2 tells two interesting tales: First, the number of adopters of Spotify's premium offer is much larger than the number of active users of the service. Estimating and predicting these two figures are essential for various reasons, including production, logistics, marketing, and price negotiations for the ad-supported part of the business. Second, the peak of the number of new adopters (2023) is ***later*** than the peak of the number of new users (2021). In the Appendix, we prove that this is a general case and not specific to this dataset.

To model this growth, we follow Libai, Muller, and Peres (2009), who added the churn factor to the basic Bass diffusion model. Using their approach, creating the aggregate user and money curves is straightforward. They consider customer churn so that when users churn, they return to the potential customer pool, where they may later re-adopt. The model is given by[3]:

$$dx/dt = \left(p + \frac{qx}{m}\right)(m - x) - \delta x \tag{2}$$

In this Equation, p is the external force coefficient such as advertising, q is the internal force coefficient, such as word of mouth and other contagion mechanisms or network effects, m is the market potential, and δ is the churn rate. In the Appendix, we replicate the results of Libai, Muller, and Peres (2009) and show that this model is equivalent to the Bass model with revised parameters, as given in Equations 2–7 in this appendix.

4. The growth and composition of users

In this section we establish several effects regarding the impact of churn on user growth, using analytical proofs and simulations. In addition, we demonstrate the effect size of the impact of churn on the users via a selected set of subscription service firms (see Web Appendix for the data sources for these firms).

4.1. The users' potential

Given that user growth can be described with a Bass-type process as per the Appendix, it follows that the user curve is S-shaped or concave, where the asymptote of the curve is the "market potential" in the classical diffusion nomenclature. In the subscription service growth model, the number of users will increase with time until it reaches an asymptotic equilibrium in which the number of adopters from the remaining potential pool equals the number of users that churn. We term this potential the "user potential," corresponding to the maximum number of users. As an example, consider the growth of the number of subscribers of SiriusXM Satellite Radio in the last 20 years in Fig. 3.

There are three key observations regarding user potential using the example of SiriusXM Satellite Radio:

First, the user potential is the user base – $x(t)$, that is s-shaped, yet it can decline temporarily. This is an essential difference from the classic economy with sales of durables. Different reasons, such as a recession, can affect current sales, but it cannot affect cumulative adoption by definition: Cumulative adoption $x(t)$ of durables adds all previous adopters and thus cannot decline. In the subscription framework, $x(t)$ is the user base, that can decline for several reasons, such as an economic downturn, a sudden increase in churn, or just bad publicity about the brand. Note that we have modeled a constant churn for simplicity, but a dynamic churn and a dummy variable for the economic conditions could be added to the Equation.[4] In Fig. 3, we see two such occurrences in 2008–2009 and the current decline that began at 2021. The likely explanations to these declines are that satellite radio is highly correlated with car ownership, and in these periods, car sales and ownership were negatively affected by external events, and increased competition from music streaming subscriptions such as Spotify.

Second, as shown from Fig. 3, users of SiriusXM have hovered just below 35m in the last four years. Just eyeballing the figure, this seems to be the effective user potential of SiriusXM. Indeed, if we estimate the model given by Equation 2 of Appendix on SiriusXM data, the users' potential $\bar{m}$ can be computed to be 35m users. Of course, many more cars are registered in the US (about 290 million), but what the data reveal is that SiriusXM has reached its user potential at around 35 m, or equivalently, that only one in eight vehicles is ever likely to have a satellite radio installed and active.

[2] The churn rate is based on Statista's estimate as it does not appear in Spotify's annual report – see our discussion on this point in the last section.

[3] The model we use is a slightly modified version of the one by Libai, Muller, and Peres (2009) without the term (1-δ) in the contagion coefficient q. The reason we can use the simplified version is that, as the authors show in the appendix to their paper, the models with and without this term are precisely equivalent.

[4] We examine the effects of dynamic churn in the discussion section.

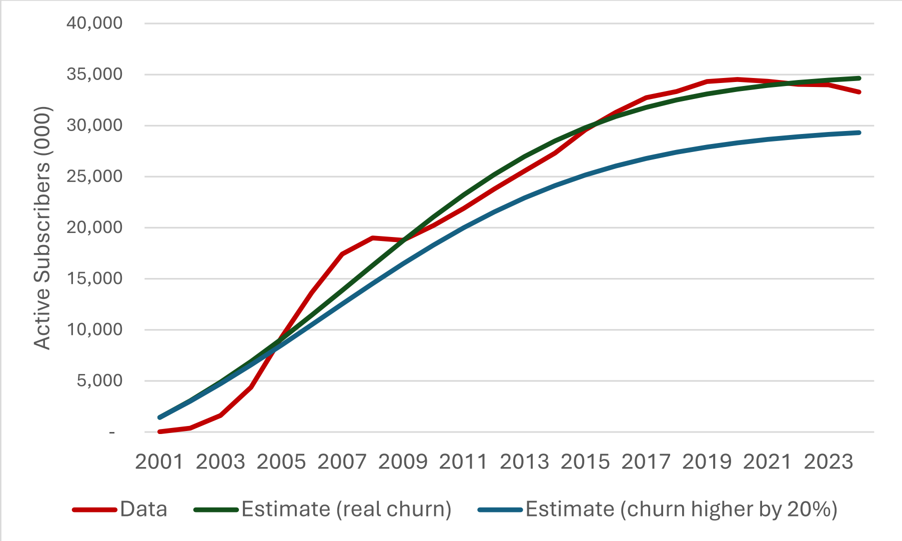

Fig. 3. SiriusXM Satellite Radio users (active subscribers in thousands) (Source: Annual reports of XM, Sirius, and SiriusXM).

Third, while the actual annual churn rate in the case of SiriusXM is 17.6 %, what would happen if it increased by 20 %, to 21.1 %, ceteris paribus? As shown in Fig. 3, the churn rate determines the size of the user potential compared to the market potential for durables with the same growth parameters. If the churn rate is zero, the user potential is the market potential of all users. As the churn rate grows, the market potential will not be realized, and the asymptote will be in a lower magnitude. This general case is summarized next, and proven analytically in the Appendix.

Effect 1: *User potential declines with an increase in churn. The effect size of this decline is demonstrated in* Table 3 *for several selected subscription service firms, ranging from 2 % to 15 % decline due to a 20 % increase in churn.*

The cost of customer churn has been traditionally attributed to its impact on the individual CLV. Hogan, Lemon, and Libai (2003) argued that the effect of churn on customer equity is due to the slowing of growth and the value of money over time. Here, we highlight that the market potential itself will be affected as some potential customers will never be seen.

This result has significant implications for the firm's growth and financial performance, as shown in the next section. In Fig. 3, we present the hypothetical case of an increase in the churn rate of SiriusXM from the current 17.6 % to about 21 %. This increase of 20 % would result in a decrease of about 15 % in the user potential, from 35m to 30m, which is a significant drop. To get a better intuition for the general case, Table 3 summarizes how a 20 % change in churn rate affects user potential in selected subscription service firms.

4.2. The peak of net users

The equivalent of the new adopter curve is the net user curve, which represents the number of new adopters minus churning customers. It is straightforward to see that the net user curve is also bell-shaped, like the new adopter curve. However, the exact shape will differ. To see that, we examine the peak of the net user curve. The sales peak is a key performance measure for firms, particularly the time to peak and its size (Fischer et al. 2010). However, while the peak in the classic adopter curve has been studied (Mahajan et al. 1990), this is not the case for the peak of net users. Consider Fig. 4, in which we observe the growth curves of net users for various churn rates.

We observe a decline in the peak of net users and an increase in the time at which this peak is achieved. The significant drop in the peak number of new users in Fig. 4 is driven by two forces: First, a decline in the fraction of new users when churn increases, and second, a corresponding decrease in the user potential. However, this can be reversed with early skewed growth in the fraction of new users, as depicted in Fig. 5.[5]

Fig. 5 tells a counterintuitive story: For early skewed growth patterns, higher churn leads to an earlier peak. The reason is that the effective external and internal coefficients are functions of churn. In the context of symmetric growth, the external parameter slightly increases with churn while the contagion coefficient sharply declines, thus leading to the pattern of Fig. 4. However, with a large external coefficient necessary for early skewed growth, this pattern reverses, and the contagion coefficient slightly declines. Consequently, when the external coefficient sharply increases, we observe the pattern shown in Fig. 5. We summarize these findings by the following effect, proven via a full factorial simulation (see Web Appendix). For the simulations we use the following parameter ranges: Diffusion parameters in the range $0.001 \leq p \leq 0.07$, and

[5] We refer to a curve to be early skewed if its peak is earlier than a symmetric function (counterintuitively called right skewed in statistics).

Table 3
The effect of churn rate on user potential in selected subscription service firms*.

Subscription service firm	Current churn rate	Current user potential (million users)	New churn rate (20 % higher)	New user potential (million users)	Percent decline in user potential
Dropbox (paid)	8.5 %	18.8	10.2 %	18.0	4 %
Peloton	15.0 %	3.0	18.1 %	2.9	2 %
SiriusXM	17.6 %	35	21.1 %	30	15 %
Spotify (premium)	17.9 %	315	21.4 %	285	10 %

* Sources: Annual and quarterly reports of the firms up to Q4 2024, and Statista (see Web Appendix).

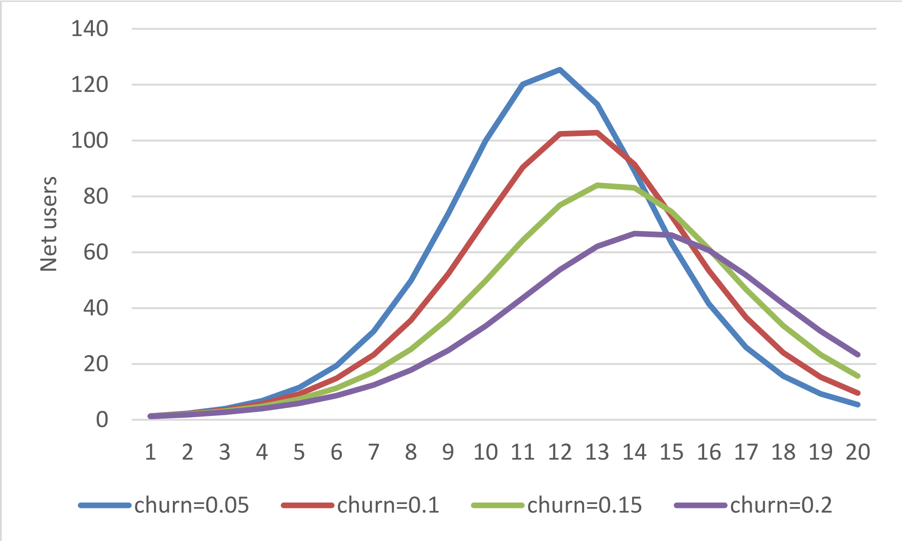

Fig. 4. Growth of net users for various churn rates (Source: Simulations of Equation 2: *p=0.01, q=0.6, m=1,000*).

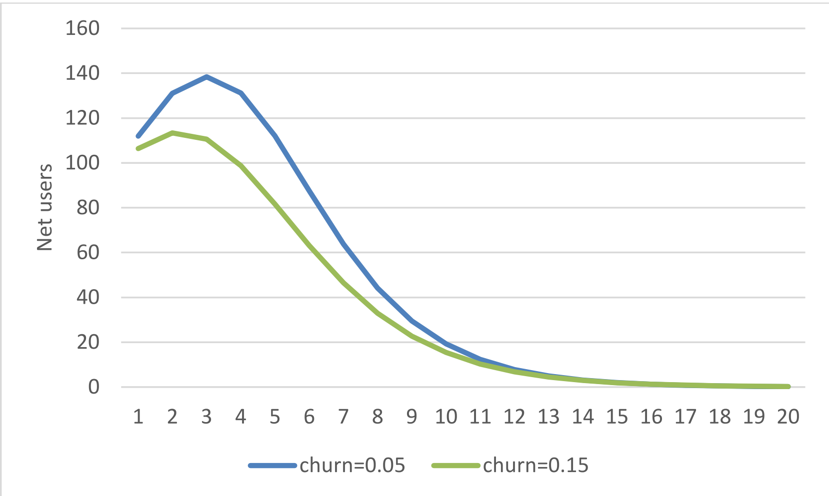

Fig. 5. Early skewed growth of net users for two churn rates (Source: Simulations of Equation 2: *p=0.1, q=0.4, m=1,000*).

$0.2 \le q \le 0.8$, consistent with the ranges observed in the new product literature. We define the churn rate range of $0.05 \le \delta \le 0.3$. We run all simulations using 20 periods.

Effect 2: *With increasing churn, the peak in the number of net users decreases, while the time to peak generally increases. However, for early skewed growth, the time to peak might decrease. The effect size of the decrease in peak users is demonstrated in* Table 4 *for a number of selected subscription service firms, ranging from a 5 % to 17 % decline as a result of a 20 % increase in churn.*

One should note that the effect size of the delayed peak as a result of an increase in churn in the four subscription service firms of Tables 2 and 3 is small. Of the four firms, only one peak changed due to a 20 % increase in churn: SiriusXM, in which, as we predicted, because of an early skewed growth (peaking already at 2008), the peak occurred one year earlier. For all the rest no change in the peak time occurs due to a 20 % increase in churn (nor for a 25 % increase).

4.3. Composition of users – Adopter categories

Considering the focus of traditional new product frameworks on first adoption, the shift to subscription thinking might require updating the fundamental thoughts on the diffusion of innovations. An important element is the adopter categories, often used for segmentation and serving as an integral part of marketing textbooks. While the traditional breakdown of adopters and innovators, early and late majority and laggards are theoretically as well as empirically supported (Appel and Muller 2021; Mahajan et al. 1990; Rogers, 2003), current thinking leans towards a simpler yet managerially relevant segmentation of just two segments: early and main market (Lehmann and Esteban-Bravo 2006; Muller and Yogev, 2006), as depicted in Fig. 6.

In the subscription context, we can ask an analogous question: What is the proportion of users belonging to each segment based on the first time they used the service? We use the Appel and Muller (2021) and Mahajan, Muller, and Srivastava (1990) frameworks and define the innovators and early adopters as the early market while the majority (early and late) as the main market. Both are based on the two inflection points of the diffusion curve: Early market size is the area under the growth curve from zero to the first inflection point, while the main market size is the area from the first to the second inflection point. We find that with an increase in churn, the relative size of the early market declines while the relative size of the main market increases, summarized by the third effect (see Appendix):

Effect 3: *With an increase in churn, the relative size of the early market declines, while the relative size of the main market increases. These effects are small, as demonstrated in* Table 5 *for several selected subscription service firms, ranging from less than 1 % to 6 % as a result of a 20 % increase in churn.*

The fact that this categorization is rather stable is not surprising given that earlier findings found the categories to be stable over time. For example, in the five decades since the 70′s, Appel and Muller (2021) found the main market to vary between 58.2 % and 60.6 %, a remarkably narrow range. The intuition behind the result is based on the growth curves in Fig. 4: High churn rate patterns are not only late in achieving the peak, but also the second inflection point is later, causing the main market to increase. Thus, analyses of segmentation predictions and optimal market behavior based on first-adoption thinking should be re-considered for the case of the growth of subscription service firms.

5. The money curve

5.1. Recurring Revenues: ARR and NRR

A notable indication of the growing importance of the money curve is the shift of firms to report Net Revenue Retention (a.k.a., Net Dollar Retention) instead of the measurement of customer retention. NRR represents the rate of change of the amount of revenue from recurring customers in a period. Specifically, the basic quantity to measure is Annual Recurring Revenues (ARR), which is the number of paying customers times their average revenue per customer. NRR is the relative change of this quantity. To compute it, we start with the latest ARR, add downgrading, upgrading, and loss due to customer churn, and then divide by ARR (see Ofek et al., 2024). The business literature illustrates NRR's emerging pivotal role, describing it as the "one metric to rule them all" and taking center stage as the "qualifying metric for determining the health of a SaaS business" (Tsang, 2022). Firms are expected to report this measure to get funding, typically hoping to reach benchmarks above 100 % (Heymann, 2023).

The shift to recurring revenues symbolizes a significant development in business thinking. Customer retention has been recognized as a critical customer-related metric and the basis of numerous research studies (Ascarza et al., 2018). However, while customer retention is still a part of NRR, the recent metric demonstrates that the customer takes a front seat in customer-related measurement for growth. It indicates an industry shift from the more straightforward user curve measurement to the more holistic money curve.

In this section, we examine two aspects of the money curve and the impact of the churn rate on them: the long vs. short-range approach and the effect of CAC on the shape of the net money curve.

5.2. The measures of money growth

There are two essential industry methods to evaluate monetary creation over time. The Annual Recurring Revenue (ARR) approach considers the revenue changes for recurring customers from one period to the next. In essence, the firm's revenue is expected to continue in the future (Liberto, 2022; Palmer, 2021). More precisely, requiring additional data and analysis, we will consider recurring profit, which accounts for the costs to serve the customer, not just revenue. However, to align with the industry terminology, we will continue to label this as ARR, and even though the time unit can be monthly rather than annual, we refer to this approach as the ARR approach.

Table 4
Churn effect on the peak in the number of users in selected subscription service firms*.

Subscription service firm	Current churn rate	Users peak (million users)	New churn rate (20 % higher)	New user peak (million users)	Percent decline in user peak
Dropbox	8.5 %	2.01	10.2 %	1.86	7 %
Peloton	15.0 %	1.02	18.1 %	0.96	5 %
SiriusXM	17.6 %	2.45	21.1 %	2.1	14 %
Spotify (premium)	17.9 %	30.7	21.4 %	25.6	17 %

* Sources: Annual and quarterly reports of the firms up to Q4 2024, and Statista (see Web Appendix).

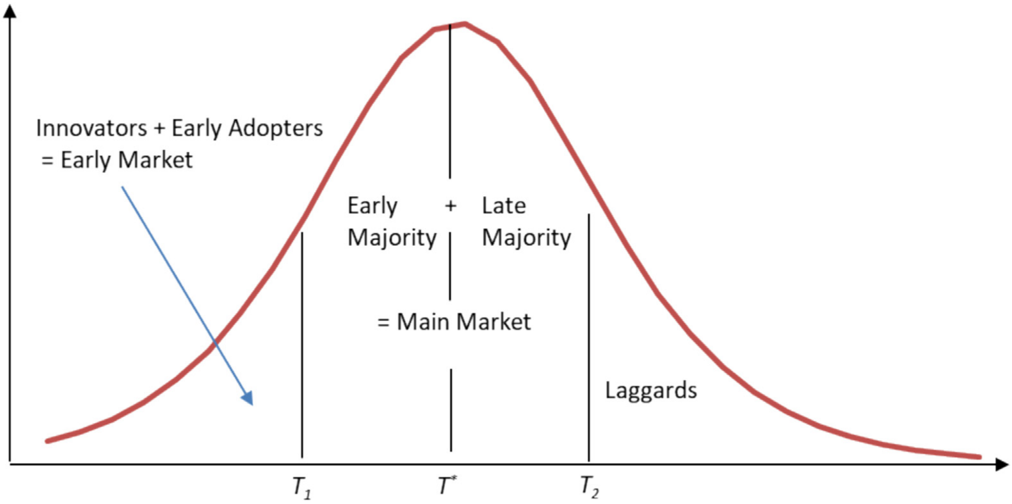

Fig. 6. Adopter Categories.

Table 5
The effect of churn on the size of the main market in selected subscription service firms*.

Subscription service firm	Current churn rate	Main Market	New churn rate (20 % higher)	New Main Market	Percent increase in Main Market
Dropbox	8.5 %	60.7 %	10.2 %	61.0 %	0.4 %
Peloton	15.0 %	57.9 %	18.1 %	57.9 %	0.01 %
SiriusXM	17.6 %	70.1 %	21.1 %	74.9 %	6.3 %
Spotify (premium)	17.9 %	60.6 %	21.4 %	61.2 %	1.0 %

* Sources: Annual and quarterly reports of the firms up to Q4 2024, and Statista (see Web Appendix).

The alternative, long-term view, which can be labeled as the Customer Lifetime Value (CLV) approach, looks at how customers are expected to affect the monetary cash stream in the long run. It is used, for example, to assess customer equity via the adoption curve. For each new adopter, the customer lifetime value is estimated at the point of adoption or averaged across a cohort in the discrete version (Gupta et al., 2004; Libai et al., 2009). As adopters increase, the monetary curve shows the accumulation of long-term value over time.

Despite their differences, both approaches underscore the importance of integrating the adoption and user curve into a money curve. Scholars advocate the CLV approach, which supports optimal firm decision-making and is consistent with stock market valuation (Schulze et al., 2012; Skiera and Schulze, 2014). Conversely, investors and practitioners often prefer the ARR approach, especially finance executives, who are concerned about resource availability and may hesitate to measure success based on yet-to-materialize long-term indicators (McCarthy et al., 2017). The business literature discusses both measures, yet it generally falls short of demonstrating and discussing the biases and disparate results that can arise from using different methods (Sacks and Ruby, 2021). To compare the two approaches in shaping the money curve, we can examine how they create customer equity in the long run.

5.3. Customer equity under the two approaches: ARR and CLV

When comparing the two approaches, we can first assess if they differ in terms of the customer equity created in the long run. Consistent with our previous notations, let $x(t)$ be the number of users at time t, da/dt– the number of new adopters at period t, g is the margin per customer per period, i denotes the discount rate, CAC is the customer acquisition costs and δ is the churn rate. We use a formulation of $CLV = g/(i + \delta)$ as in the standard approach.

The CLV method (Equation 3) takes the number of ***new adopters*** at each period and multiplies it by the CLV of each new adopter minus the cost of acquiring this user. Thus, it is

$$\text{CLV Based Customer Equity} = \int_0^\infty (CLV - CAC) \cdot \frac{da}{dt} \cdot e^{-it} dt \tag{3}$$

The ARR approach (Equation 4) takes the firm's number of users and multiplies each by the unit gross profit margin of the subscription service minus the cost of acquiring the new adopters (CAC). As in the case of the CLV approach, to derive customer equity, one computes the NPV of these streams using the firm's cost of capital.

$$\text{ARR Based Customer Equity} = \int_0^\infty \left(g \cdot x(t) - CAC \cdot \frac{da}{dt} \right) \cdot e^{-it} dt \tag{4}$$

Intuitively, we should get to the same infinite customer equity in both cases and show it formally in the Appendix. This is because the underlying data in both cases is a matrix representing the revenues (or profits) for each cohort in each time period. If we sum the rows, we obtain the cohort-by-cohort view. If we sum the columns, we acquire the period-by-period view. If the underlying matrix is the same, both views must yield the same valuation (for more on this, see McCarthy et al., 2017; Skiera and Schulze, 2014).

The practical question is still what happens with a finite shorter horizon. Managers may look forward only for a couple of years, even when they look at CLV created, because of the uncertainty associated with longer-term financial horizons (Kumar and Shah, 2008). Thus, they can look at the ARR created in a finite number of years or the customer equity during that period. Because the CLV approach captures a larger portion of the infinite customer equity, it will be closer to the infinite customer equity, thereby more accurately reflecting the true customer equity (see Appendix for the analytical proof). The difference between the two approaches is demonstrated in Table 6.

Several factors can affect the difference between CLV and ARR, but as before, we look at customer churn. Customer churn will affect both the ARR and the CLV finite horizon approach. However, when simulating the two scenarios, the effect on the CLV approach is larger because it impacts a larger number of years captured in the CLV. Hence, when customers churn more, the difference in the finite case is lower between the two approaches.

Effect 4: *While ARR and CLV methods of customer equity evaluation yield the same value in the long-run, with a finite time horizon, the CLV method more accurately reflects the true value of customer equity. The difference decreases with an increase in churn. The effect size is demonstrated via simulations in* Table 6.

5.4. The net money growth

Our final question concerns the shape of the money curve. Standard industry recurring revenue analysis for subscription firms excludes customer acquisition costs (Paddle, 2023). However, these costs can be a significant burden on the cash stream. We are interested in the shape of the ''net money curve,'' which takes CAC into account.

In the case of the CLV approach, we can expect that the shape of the money curve will resemble that of the adoption curve. We can multiply the cohort size by (CLV-CAC) for each entering cohort to get the monetary profits associated with this cohort. Since investors and managers demand CLV > CAC, this number will typically be positive and will change the curve's magnitude.

In the case of the ARR curve, the stream of profits for an individual customer is spread over time, and the CAC, which may be much larger than the first period of profit from a customer, precedes it. The firm can have a ''trough'' in which cash flows are negative. Thus, the ''net money curve'' shape for ARR can be very different from the user curve.

This cash flow trough can have severe consequences for new subscription service firms that need financial support from investors with limited resources and are struggling to demonstrate profitability (Skok, 2017). In such cases, the more successful the marketing efforts, the faster the growth, which in turn amplifies the problem. Managers are often not aware of how significantly more rapid growth can deepen the trough (Skok, 2017). Next, we will examine how the speed of growth and the churn rate influence the trough and, consequently, the shape of the money curve.

Consider the example of a new subscription business with customer acquisition costs of \$60, and an annual subscription margin of approximately \$20. With an annual churn rate of 15 % and a discount rate of 10 %, the CLV is \$80, leading to CLV/CAC of 1.33 (80/20). However, as it is a new service, the number of new customers increases over time, with the rate of growth captured by the diffusion parameters.

The firm contemplates two scenarios. Under the current ''low growth'' scenario, the growth parameters are $p = 0.001$ and $q = 0.6$. If the marketing department successfully boosts growth to a ''high growth'' scenario, the firm anticipates a 20 % increase in both p and q. Ignoring customer acquisition costs and using the continuous recurring consumption model, we

Table 6
The effect of churn on customer equity for a five-year horizon*.

	Customer Equity	Customer Equity with increased churn rate (20 % higher)
CLV based Customer Equity	\$25,089	\$22,369
ARR based Customer Equity	\$11,489	\$10,854
CLV vs ARR percent change	−54 %	−52 %

* The table reports the average CLV and ARR based customer equity across all simulations. We use the same simulation parameters as specified before: $0.001 \le p \le 0.07$; $0.2 \le q \le 0.8$; $0.05 \le \delta \le 0.3$, g = 20, i = 0.1.

can observe the recurring margin over time for both scenarios in Fig. 7a. As expected, the high growth scenario appears more favorable.

However, the picture changes when customer acquisition costs are considered (see Fig. 7b). Two basic measures of the trough are evident from this curve: Its width (cash flow begins to be positive) and its depth (what is the most negative value of the cash flow). We see that under fast growth, the positive cashflow occurs earlier, yet the depth of the trough is larger. In this case, the overall negative cashflow effect appears larger for the faster case. Of course, once it becomes positive, the high growth case would lead to a faster-rising money curve, and we can expect that in the long run, customer equity will be higher.

The pattern of the trough is thus affected by two growth tradeoffs. First, under higher growth scenarios, the firm incurs higher CAC early on, yet it also accumulates paying customers more quickly, which accelerates the return to profitability. Second, we can expect another tradeoff due to the churn rate. Higher churn implies that more time is needed for the acquired customers to bring the firm to profitability. Conversely, as noted earlier, high churn results in slower growth, thus reducing expenditure on CAC. To generalize this observation, we ran a simulation where we changed the various profitability and growth parameters in the previously defined ranges (see Web Appendix). We demonstrate via this simulation that faster customer acquisition increases the magnitude of the cashflow through. Hence, the following effect emerges:

Effect 5: *With an increase in churn, the cash flow trough, that is the maximum negative cash flow of a new venture, increases. These effects can be substantial, as demonstrated in Table 7 for several industries, ranging from 6 % to 26 % as a result of a 50 % increase in churn.*

To understand the magnitude of the impact of churn on the maximum negative cashflow across different industries, we collected data on churn rate, average CAC, and average lifetime value for subscribers in selected industries (see Web Appendix, Table 1). These data enabled us to assess the effect of changes in the churn rate on the cash flow trough. Table 7 demonstrates the effect of a 50 % increase in churn rate on the trough, measured per 1,000 people. The effects can be substantial, however, non-linear: in four out of five cases, the trough increased, and in one, it decreased. Thus, the pattern of the trough and its drivers is still an open issue. The interactions of churn, growth, the ratio of CAC to per-period revenue and costs, and the level of the discount rate make the prediction nontrivial. This complexity in analyzing the money curve presents a promising research opportunity.

5.5. Covert vs. Overt churn effect

When adopters churn, there are two primary financial repercussions due to churn's influence on customer equity: The **overt direct churn effect** which pertains to the loss of cash flows from the departing individual customer. This effect is deemed 'overt' as it is immediately visible to the firm – a customer has left and is no longer paying the subscription fee. On the other hand, the **covert indirect churn effect** arises from potential customers who never join the subscription due to a diminished user base in addition to any deceleration that affects the value of money over time in the growth process (Hogan, Lemon and Libai, 2003). This effect is referred to as 'covert' as it is not readily apparent to the firm – it is not registered in the books, leaving the firm unaware of the lost potential customers.

To be able to assess and quantify the overt and covert churn effect we conduct simulations using the competitive model of Libai, Muller and Peres (2009). We simplify the model by assuming that there's churn to and from the two firms and that disadoption from the category is negligible. The model is given by Equation 5 for $i = 1, 2$, and $j \neq i$:

$$\frac{dx_i}{dt} = \left(p_i + \frac{q_i x_i}{m}\right)(m - x) - c_i x_i + c_j x_j \tag{5}$$

In this equation, p_i is the external coefficient for firm *i*, q_i is the internal coefficient for firm i, c_i is the churn rate for firm *i*, m is the common market potential, and $x = x_1 + x_2$. The logic behind the computation is as follows: Consider the case of $p_i = 0.02, q_i = 0.3, c_i = 0.2$, and the common parameters are given as $m = 2000, i = 10\%$. This is a symmetric case where within about 25 periods each firm achieves its own effective potential of 1,000, and Customer Equity of \$5,267. Now suppose Firm 1 reduces its churn by 10 %, so that $c_1 = 0.18$ while all other parameters remain unchanged. Firm 1 gained \$234 $(5,501 - 5,267)$, while Firm 2 lost \$582 $(5,267 - 4,685)$. As in Libai, Muller and Peres (2009), we assume only two competitors and no outside option (by the end of the time horizon, the entire market will have adopted the product). It thus follows that Firm 1 gained what Firm 2 lost. Firm 2 lost \$582, out of which \$234 are due to the loss of customers to Firm 1, and hence the loss due to the decrease in market potential is $234/582 = 40\%$ and thus the loss due to slower diffusion is 60 %.

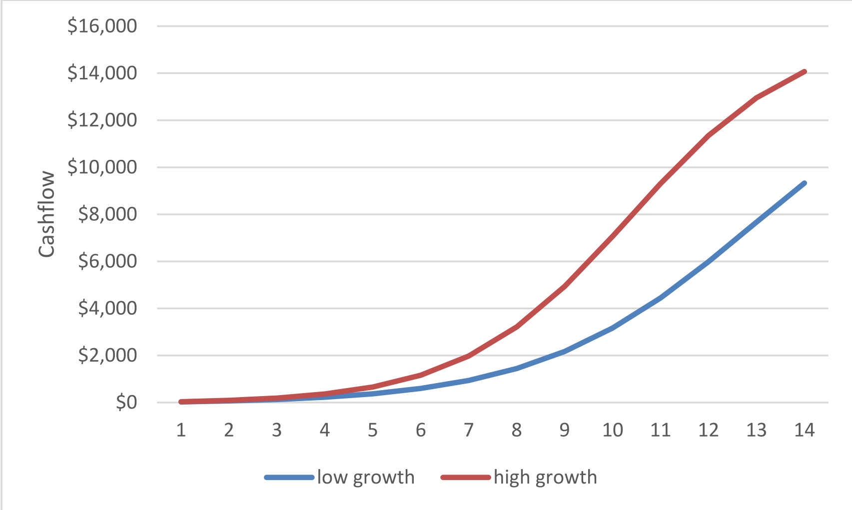

Fig. 7a. Two cashflow scenarios without customer acquisition costs (Source: Simulations, *CAC=$0, p = 0.001, q = 0.6, m = 1,000, g = $20, i = 0.1, δ = 0.15*).

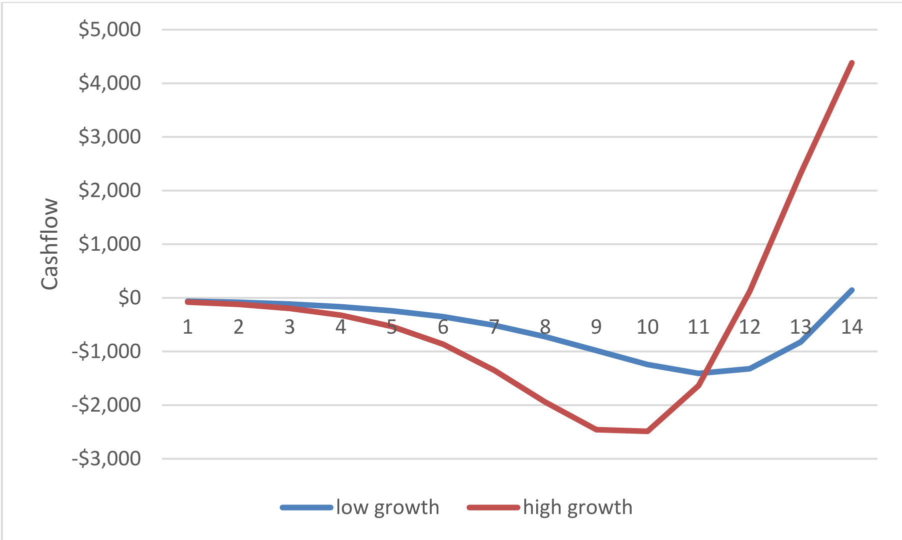

Fig. 7b. Two cashflow scenarios with customer acquisition costs (Source: Simulations, *CAC = $60, p = 0.001,q = 0.;6, m = 1,000, g = $20, i = 0.1, δ = 0.15*).

Table 7
Maximum negative cash flow (trough) per 1,000 people in selected industries.*

Industry	Current churn rate	Baseline trough per 1,000 people	New churn rate	New trough per 1,000 people	% difference
Food & beverage	8.7 %	$899	13.1 %	$663	26.3 %
Fashion & apparel	8.7 %	−$3,231	13.1 %	−$3,425	6.0 %
Health & wellness	9.9 %	−$5,425	14.9 %	−$5,940	9.5 %
Beauty & personal care	8.0 %	−$3,901	12.0 %	−$4,289	10.0 %
Home goods	8.4 %	−$3,702	12.6 %	−$4,084	10.3 %

* Sources: See Web Appendix.

To generalize the covert and overt churn effect, we run simulations using the same parameter ranges as outlined before: diffusion parameters in the range $0.001 \leq p \leq 0.07$, and $0.2 \leq q \leq 0.8$, consistent with the ranges observed in the new product literature. We define the churn rate range of $0.05 \leq \delta \leq 0.3$ and reduce in every parameter combination the churn of firm 1 by 20 % relative to firm 2.[6] Our results show that across all simulations the **overt direct churn effect** pertaining to the loss of cash flows is 46 % on average, and the **covert indirect churn effect** that arises from potential customers who never join the subscription service due to a diminished user base, amounts to 54 % of the total loss on average.

Effect 6: *The covert indirect effect of an increase in churn, that refers to the loss of potential customers who never join the subscription service due to a diminished user base, can be larger than the overt direct churn effect represented by the loss of direct cash flows. Effect size, found via simulations, indicates that the covert churn effect amounts to about 54 % of the total loss of an increased churn.*

6. Implications for a growth-oriented subscription service firm

A service mindset is an essential part of modern marketing and, in a broader sense, one of the fundamental ideas that govern contemporary business (Huang and Rust, 2021; Rust, 2020; Vargo and Lusch, 2004) and thus, subscription service growth thinking is fundamental to current new product management. As we highlighted above, customer churn plays a critical role in this context. Next, we discuss further implications that stem from the role of churn in new subscription growth.

6.1. Reporting growth metrics

Of the four subscription service firms we report in Tables 3-5, namely, Dropbox, Peloton, SiriusXM and Spotify, all reported the number of users, under various definitions such as subscribers, active accounts, or monthly active users. Yet, Peloton and SiriusXM report the churn rate, while Dropbox and Spotify do not. The fact that the regulatory agencies in the US and Europe do not require firms to report churn is quite striking: Churn is a key figure indicating a subscription-based firm's operational and marketing health and expected future performance.

Moreover, academics across disciplines argue that without informative customer measures, current financial disclosure rules allow would-be public companies to shape a rosy narrative about their prospects and disguise information that investors should know (Damodaran et al., 2022). Similar sentiments about the need to report customer-based measures to conduct proper valuation come from consulting firms (Markey, 2020) and executives (Brennan, 2020). If the number of products sold in previous times may have been sufficient to value growth, the dominance of service-based growth requires more compound measures. Indeed, academics have developed ways to figure out the fundamental customer metrics from traditional financial reporting to enable valuation (McCarthy, Fader, and Hardie 2017). Like others, we believe it is time for regulators to adapt.

6.2. The growth-profitability balance

The relevance and significance of the topics addressed above are reflected in a critical question that investors and managers of new subscription service ventures grapple with: the balance between growth and profitability. Prioritization of either approach has been revised multiple times, with no definite conclusion reached. While several years ago, managers were urged to prioritize growth at the expense of customer profitability (Mankins, 2017), more recently, the focus has shifted toward customer profitability (Harrington, 2022; Heim, 2022), but reports suggest a possible return to prioritizing growth (Wilhelm, 2023).

This growth-profitability conundrum is fundamentally tied to assumptions about the benefits of early market capture and its long-term impact on profitability. Nonetheless, the measurement and analytical structures must allow for comparability, transparency, and comprehensive quantitative analyses. We propose that the subscription growth framework offers an appropriate blueprint. The central issue is maximizing customer equity, with different strategies that could rely on either the Annual Recurring Revenues (ARR) or Customer Lifetime Value (CLV), rooted in the adoption and user curve.

Within this context, we should acknowledge the financial limitations of new ventures and the reality of scarce resources, which may impede the implementation of an optimal strategy. While a CLV-centric approach aligns with long-term economic planning, recurring metrics like ARR cater to investors' interest in short-term analyses. This consideration is particularly critical given new ventures' limited resources and the need to demonstrate product-market fit to secure further funding (Georgiadis, 2023). Digital insurance venture Lemonade serves as a pertinent case for this point. Lemonade reports a unit economics (CLV/CAC) ratio of three, indicating a strong standing from a CLV perspective. However, customer acquisition costs still pose a significant short-term challenge. To offset this liability, Lemonade partnered with an investment firm that now covers 80 % of customer acquisition costs in exchange for a future return from the customers' lifetime value (Lemonade, 2023).

[6] We exclude cases where very low churn leads to negative acceleration (12% of cases when churn of firm 1 is in the range of 0.056 to 0.080).

Analyzing the benefits of such a scenario for Lemonade or any similar service growth firm requires an in-depth understanding of the three growth curves – adoption, users, and money. The role of churn in all three curves further highlights its importance in also guiding the growth-profitability balance.

6.3. Dynamic churn

An additional issue concerns the stability of the churn rate. For example, recent research into the growth of subscription firms suggests that a) there is cross-cohort heterogeneity that can affect the churn rate over time, and b) the overall churn rate tends to decline over time, regardless of cross-cohort differences due to the heterogeneity among customers at the cohort level (McCarthy et al., 2024). We assumed a stable churn rate over time, while in practice, it may vary in specific situations (Ascarza et al., 2018).

Following the modeling frame, we build on, dynamic churn has to be modeled carefully. If the change in churn affects all cohorts in the user base equally, that is, they all have the same churn at each period t, then Equation 2 still holds with $\delta(t)$ replacing δ. However, if each cohort's churn differs from other cohorts, then Equation 2 should be replaced with the following integral equation, which cannot be reduced to a differential equation:

$$x(t) = \int_0^t I(s)e^{-\delta(s)(t-s)}ds \tag{6}$$

$$\text{where } I(s) = (p + qx(s)/m)(m - x(s))$$

To see this more clearly, consider a firm with a stock of machines such as printers that it rents. These machines decay at the date of $\delta(t)$ for each batch that was produced at time period t. This decay rate is the analogy of churn in our setting. If the firm now decides to replace the stainless-steel ball-bearing with plastic ones, this will affect the decay of the current batch, but none of the previous batches, as the latter still use the steel bearing. Thus, each batch (cohort) decays at its own rate; consequently, Equation 5 is appropriate (Muller and Peles, 1990). There are good reasons to believe that subscription service firms' cross-cohort heterogeneity will be reflected in different churn rates for different cohorts.

6.4. Free version and freemium

All of the four subscription service firms we report in Tables 3-5 (Dropbox, Peloton, SiriusXM and Spotify) offer a free version. While Dropbox and Peloton offer a scaled down version for free, unlimited in time, SiriusXM, in conjunction with car manufacturers, offers the service for free for the first three months of the purchase of the vehicle. Spotify's free version is advertising based, and thus free only in the sense of out-of-pocket expenses, but requires the user to listen to ads. Similarly, in the freemium product market, a popular business model among mobile apps, and particularly games, only a subset of users pays for the product. Non-paying users, however, may still generate revenue through advertising or in-app purchases. The freemium model has been the basis for numerous studies exploring optimal customer management (Deng et al., 2023; Shi et al., 2019).

In such markets, the service life cycle becomes more intricate due to the presence of two distinct user groups – paying and non-paying users – and the interactions between them: The decision on the difference between the offers to the two segments affects growth of these two segments directly and indirectly via the churn rate of each. For example, in Spotify's free version, users typically encounter ads (of 15–30 s in length) after every 2 to 3 songs. Now suppose Sportify chooses to increase the level of ads to one every song instead of every 2 to 3 songs. This will affect not only the churn rate of the ad-supported segment of Spotify but also reduce the attractiveness of the free offer at its two extremes: It will make it less attractive to nonusers, and will increase the conversion rate from the free to the paid versions. As the free version typically acts as a gateway to the much more profitable paid version, this will have a direct implication on the future growth and profitability of the service.

6.5. Lost for good effect

One possible way to model churn behavior is the "Lost for Good" case, where churners do not return to the adopter's pool, but leave the firm's potential pool for good. One should note that lost for good is an abstraction of a boundary case, where the real issue is the difficulty of winning back a lost customer. Thus, we can define a lost for good effect to be larger, when the consumer's likelihood to return to the potential pool is smaller. The competitive framework presented in the previous section allows us to investigate the consequences of the lost for good effect. We start with two competitors experiencing equal churn, meaning the churn to and from the focal firm (Firm 1) is equal. Then, we reduce the incoming churn from Firm 2. This gradually accelerates the lost for good effect as the users who leave Firm 1 become less and less likely to come back. When the churn from Firm 2 is zero it becomes the classical lost for good case. This analysis is demonstrated in Fig. 8 (for $p = 0.03$, $q = 0.5$). One can observe a two-fold result of the lost for good effect: First, customer equity of the focal firm (Firm 1) declines, and it does so at an increasing rate. Thus, what we have demonstrated in our paper is a conservative effect of churn that is likely larger when the lost for good effect is larger.

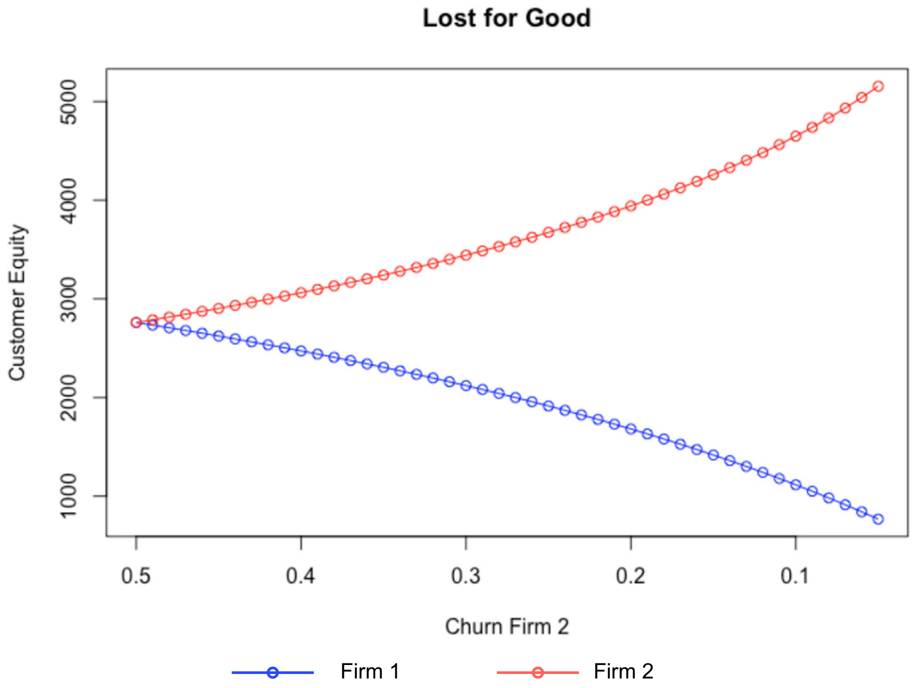

Fig. 8. Customer Equity with lost for good effect.

6.6. Advertising and pricing in subscription service growth

Advertising and pricing have been incorporated successfully into the diffusion of innovation framework; however, these efforts are generally focused on first-purchase scenarios (see Bass et al., 2000). These marketing mix variables were modeled to affect the growth parameters or the market potential, and dynamic optimal policies were identified using the extended Bass model as the basic framework (Cosguner and Seetharaman, 2022). These extended models and corresponding optimal policies could be replicated for the service life cycle framework as well.

Beyond the contribution to improved fit and prediction (Mesak et al., 2022), various first-purchase marketing mix growth insights should be examined for the case of growing recurring services. For example, findings from studies on customer purchase behavior for subscriptions (Iyengar et al., 2022) and service pricing optimization (Wang et al., 2019) can be used to guide thinking on how pricing will change throughout the growth of subscription services. Notably, the effects of churn on the optimal path of advertising and pricing offer further managerially relevant questions: For example, should an increase in churn increase or decrease optimal advertising levels? It should be noted that acquiring new customers via any subsidy or seeding campaign only accelerates their acquisition. If the market potential is correctly defined, these customers would be otherwise acquired sometime in the future. Therefore, not their entire CLV should be added as a benefit of such action, but just the benefit of having their CLV earlier (Libai et al., 2013).

7. Conclusion

The modeling of service growth enabled researchers and managers to assess customer equity and the value of growing recurring service firms. The next step is to apply these models to gain a broader perspective on new service growth. We need to extend the rich literature on the diffusion of innovations, which has been largely based on first-purchase analysis, to the case of new services. To do so, we must consider three curves: adoption, users, and money.

To demonstrate the potential for this analysis, we focused on customer churn and its impact on the growth of new subscription services. Building on the Libai, Muller and Peres (2009) modeling framework, we demonstrated how churn can affect classic diffusion issues of interest: The peak of adopters and users, market potential, and adopter growth categories. We further demonstrated that churn could help us understand the transformation from the user curve to the money curve. We pointed to different implications of this approach, including issues of reporting, the growth-profitability balance, and the need to consider the covert effect of churn. These aspects should, of course, be further explored. Given the significant role of services among new products of innovation, we hope these insights can help motivate researchers to extend the classic new product growth literature further, delve deeper into the dynamics of new service growth, and the impact of factors like customer churn.

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CRediT authorship contribution statement

Barak Libai: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Eitan Muller:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Verena Schoenmueller:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

This appendix proves Effects 1, 3, and 4. The model is a simpler version of Libai, Muller and Peres (2009) without the term $(1-\delta)$ in the contagion coefficient:

$$\frac{dx}{dt} = \left(p + \frac{qx}{m}\right)(m - x) - \delta x \tag{1}$$

Libai Muller and Peres showed that this model is equivalent to the Bass model with the following new parameters, that is, the solution of Equation1 is given by Equation 2:

$$x(t) = \bar{m} \cdot F(t) = \bar{m} \cdot \frac{1 - e^{-(\bar{p}+\bar{q})t}}{1 + \left(\frac{\bar{q}}{\bar{p}}\right) \cdot e^{-(p^- + q^-)t}} = \bar{m} \cdot \frac{1 - e^{-\Delta t}}{1 + \frac{\Delta+\beta}{\Delta-\beta} \cdot e^{-\Delta t}} \tag{2}$$

Where,

$$\bar{p} = (\Delta - \beta)/2 \tag{3}$$

$$\bar{q} = (\Delta + \beta)/2 \tag{4}$$

$$\bar{m} = m(\Delta + \beta)/(2q) \tag{5}$$

$$\beta = q - p - \delta \tag{6}$$

$$\Delta = \sqrt{\beta^2 + 4qp} \tag{7}$$

Effect 1: When churn (δ) increases, user potential $(\bar{m})$ declines, that is $\frac{\partial \bar{m}}{\partial \delta} \le 0$.

Proof: First note that given Equation 6, it follows that $\frac{\partial \bar{m}}{\partial \delta} = -\frac{\partial \bar{m}}{\partial \beta}$. Next, it's straightforward to show that:

$$\frac{\partial \Delta}{\partial \beta} = \frac{\beta}{\Delta} \tag{8}$$

Using Equations 5 and 8 yields the following:

$$\frac{\partial \bar{m}}{\partial \delta} = -\frac{\partial \bar{m}}{\partial \beta} = -\frac{m}{2q} \cdot \left(1 + \frac{\beta}{\Delta}\right) \leq 0 \tag{9}$$

Note that we will shortly constrain β to be nonnegative (for the peak time T^* to be nonnegative), yet here we need a weaker condition, $\bar{q} \geq 0$.

We also with to show that the peak in the number of new adopters is later than the peak in the number of new users. Recall that the relation between the number of new adopters to the number of new users is given by:

$$da/dt = dx/dt + \delta x \tag{10}$$

Differentiate Equation 10 to verify that when $d^2x/dt^2 = 0$, $d^2a/dt^2 = \delta dx/dt > 0$.

Effect 3: With an increase in churn, the relative size of the early market declines, while the relative size of main market increases.

Proof: We use the Appel and Muller (2021) and Mahajan, Muller and Srivastava (1990) frameworks and define the innovators and early adopters as early market while the majority (early and late) as main market.

The critical points, that is inflection points T_1 and T_2 and peak-time T^* are given by (see Mahajan Muller and Srivastava 1990, and Fig. 6 in the main text):

$$T_{1,2} = +\frac{1}{\bar{p} + \bar{q}} \ln\left(\frac{\bar{q}}{\bar{p} \cdot \left(2 + \sqrt{3}\right)}\right) = T^* \mp \frac{1}{\Delta} \cdot \ln\left(2 + \sqrt{3}\right) \tag{11}$$

$$T^* = \frac{\ln(\bar{q} / \bar{p})}{\left(\bar{p} + \bar{q}\right)} = \frac{1}{\Delta} \cdot \ln\left(\frac{\Delta + \beta}{\Delta - \beta}\right) \tag{12}$$

For T^* to be nonnegative, we need the log in the RHS of Equation 12 to be nonnegative and thus we need $\Delta + \beta \geq \Delta - \beta$ which implies that $\beta \geq 0$. For p to be nonnegative we need $\Delta \geq \beta$, which obviously holds given Equation 7. As this appendix investigates the effects of churn δ on the adopters' categories and the timing and size of peak sales, for the rest of the analysis we assume that $T^* \geq 0$, that is, $\beta \geq 0$.

Let M be the size of the early majority. From MMS we can easily show that M is given by:

$$M = \frac{1}{\sqrt{12}} \cdot \frac{2\Delta}{\Delta + \beta} \tag{13}$$

As the sizes of the early and late majority are equal, the main market is equal to $2 \cdot M$, and thus showing that the main market increases with churn, is equivalent to showing that M increases with churn, and thus:

$$\frac{\partial M}{\partial \delta} = -\frac{\partial M}{\partial \beta} = -\frac{2}{\sqrt{12}} \cdot \frac{(\Delta + \beta) \cdot \frac{\partial \Delta}{\partial \beta} - \Delta \cdot \left(\frac{\partial \Delta}{\partial \beta} + 1\right)}{(\Delta + \beta)^2} = -\frac{2}{\sqrt{12}} \cdot \frac{\beta \cdot \frac{\partial \Delta}{\partial \beta} - \Delta}{(\Delta + \beta)^2} \tag{14}$$

Using Equation 8 we have:

$$\frac{\partial M}{\partial \delta} = -\frac{2}{\sqrt{12}} \cdot \frac{\beta^2 - \Delta^2}{\Delta(\Delta + \beta)^2} = +\frac{2}{\sqrt{12}} \cdot \frac{\Delta - \beta}{\Delta(\Delta + \beta)} \geq 0 \tag{15}$$

With strict inequality when $\Delta > \beta$, that is, when both p and q are positive.

Next, we show that the early market declines in size when δ increases. From MMS the early market category size E is given by:

$$E = \frac{1}{2} \cdot \left(1 - \frac{\Delta - \beta}{\Delta + \beta}\right) - \frac{1}{\sqrt{12}} \cdot \left(1 + \frac{\Delta - \beta}{\Delta + \beta}\right) = \frac{\beta}{\Delta + \beta} - \frac{2}{\sqrt{12}} \cdot \frac{\Delta}{\Delta + \beta} = \frac{\beta}{\Delta + \beta} - M \tag{16}$$

Since we have shown that $\partial M / \partial \delta \geq 0$ it's enough to show that the derivative of the first term in Equation 1 is declining in δ.

$$\frac{\partial \frac{\beta}{\Delta + \beta}}{\partial \delta} = -\frac{\partial \frac{\beta}{\Delta + \beta}}{\partial \beta} = -\frac{(\Delta + \beta) - \left(\frac{\partial \Delta}{\partial \beta} + 1\right)\beta}{(\Delta + \beta)^2} = -\frac{(\Delta + \beta) - \left(\frac{\beta}{\Delta} + 1\right)\beta}{(\Delta + \beta)^2} = -\frac{\Delta - \beta}{\Delta(\Delta + \beta)} \leq 0 \tag{17}$$

Effect 4: Customer Equity of a service firm is given by either the CLV method (Equation 18) or the ARR method (Equation 19). In other words, these two measures are equivalent. With a finite horizon, the CLV method yields a higher value that more accurately reflects the true customer equity.

$$\pi_{CLV} = \int_0^\infty (CLV - CAC) \cdot \frac{da}{dt} \cdot e^{-it} dt \tag{18}$$

$$\pi_{ARR} = \int_0^\infty \left(g \cdot x(t) - CAC \cdot \frac{da}{dt} \right) \cdot e^{-it} dt \tag{19}$$

Proof: With no acquisition costs, customer equity is given by:

$$\begin{aligned} \pi &= \int_0^\infty CLV \cdot \frac{da}{dt} \cdot e^{-it} dt = CLV \cdot \int_0^\infty \left\{ \frac{dx}{dt} + \delta x(t) \right\} \cdot e^{-it} dt \\ &= \frac{g}{(i+\delta)} \cdot \int_0^\infty \left\{ \frac{dx}{dt} + \delta x(t) \right\} \cdot e^{-it} dt \end{aligned} \tag{20}$$

We wish to show that π can also be written as:

$$\pi = \int_0^\infty g \cdot x(t) \cdot e^{-it} dt \tag{21}$$

The way to show it is to take the first part of Equation 20, and integrate by parts:

$$\int_0^\infty \left\{ \frac{dx}{dt} \right\} e^{-it} dt = \lim_{t\to\infty} x(t) e^{-it} - \lim_{t\to 0} x(t) e^{-it} + i \int_0^\infty x(t) e^{-it} dt = \int_0^\infty i \cdot x(t) e^{-it} dt$$

Where the last equality follows the fact that $x(\infty)$ is finite, and $x(0) = 0$. Thus:

$$\pi = \frac{g}{(i+\delta)} \cdot \int_0^\infty \left\{ \frac{dx}{dt} + \delta x(t) \right\} \cdot e^{-it} dt = \frac{g}{(i+\delta)} \cdot \int_0^\infty \{ix(t) + \delta x(t)\} \cdot e^{-it} dt = g \cdot \int_0^\infty x(t) \cdot e^{-it} dt \tag{22}$$

Adding customer acquisition costs (CAC) simply subtracts the same term $\int_0^\infty CAC \cdot \frac{da}{dt} \cdot e^{-it} dt$ from both equations.

For the finite horizon case, denote by $\pi_{CLV,T}$ the customer equity up to time horizon T, according to the CLV approach, and similarly $\pi_{ARR,T}$ according to the ARR approach. Note that we abstract from the costs of acquiring these users as the exact same costs term is subtracted from both cases, namely: $\int_0^T CAC \cdot \frac{da}{dt} \cdot e^{-it} dt$.

$$\pi_{CLV,T} = \int_0^T CLV \cdot \frac{da}{dt} \cdot e^{-it} dt \tag{23}$$

$$\pi_{ARR,T} = \int_0^T g \cdot x(t) \cdot e^{-it} dt \tag{24}$$

We now employ the same integration by parts of the CLV approach that yields the following:

$$\int_0^T \left\{ \frac{dx}{dt} \right\} e^{-it} dt = x(T) e^{-iT} - \lim_{t\to 0} x(t) e^{-it} + i \int_0^\infty x(t) e^{-it} dt = x(T) e^{-iT} + \int_0^\infty i \cdot x(t) e^{-it} dt$$

Thus,

$$\begin{aligned} \pi_{CLV,T} &= \frac{g}{(i+\delta)} \cdot \int_0^T \left\{ \frac{dx}{dt} + \delta x(t) \right\} \cdot e^{-it} dt = \frac{g x(T) e^{-iT}}{(i+\delta)} + \frac{g}{(i+\delta)} \cdot \int_0^T \{ix(t) + \delta x(t)\} \cdot e^{-it} dt \\ &= \frac{g x(T) e^{-iT}}{(i+\delta)} + g \int_0^T x(t) \cdot e^{-it} dt = CLV \cdot x(T) \cdot e^{-iT} + \int_0^T g \cdot x(t) \cdot e^{-it} dt \end{aligned} \tag{25}$$

It follows that,

$$\pi_{CLV,T} = CLV \cdot x(T) \cdot e^{-iT} + \pi_{ARR,T} \tag{26}$$

And thus,

$$\pi_{CLV,T} > \pi_{ARR,T} \tag{27}$$

As both measures undervalue the true customer equity (the one with infinite horizon), it follows that for the finite horizon, the CLV method is more accurate.

Web Appendix. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijresmar.2025.03.005.

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