A NONUNIFORM INFLUENCE INNOVATION DIFFUSION MODEL OF NEW PRODUCT ACCEPTANCE*

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A nonuniform influence (NUI) innovation diffusion model for forecasting first adoptions of a new product is proposed. An extension of the Bass model, the proposed model overcomes three limitations of the existing single-adoption diffusion models. First, the current models generally assume that the word-of-mouth effect remains constant over the entire diffusion span. However, for most innovations, the word-of-mouth effect is likely to increase, decrease or remain constant over time. Second, the existing models assume that an innovation must attain its maximum penetration rate before capturing a prespecified level of potential market, for example, 50%. That is, they restrict the location of the inflection point for the diffusion curves. Third, the current models assume that the adoption patterns after and before the location of maximum penetration rate are mirror images of each other. That is, the diffusion curve is symmetric. By allowing the word-of-mouth effect to systematically vary over time, the proposed model allows the diffusion curve to be symmetrical as well as nonsymmetrical, with the point of inflection responding to the diffusion process. Data from five consumer durables are analyzed to illustrate the generality of the model.
(Diffusion; Technological Forecasting; Word-of-Mouth)

1. Introduction

Many studies in the areas of marketing, technological forecasting and economics have attempted to model the time-dependent aspects of the innovation diffusion process (Mahajan and Muller 1979). The underlying behavioral theory in the development of these models is that the innovation is first

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adopted by a select few innovators who, in turn, influence others (via word-of-mouth) to adopt it (Rogers 1983). Examples of the best-known diffusion models in marketing include the models suggested by Bass (1969), Fourt and Woodlock (1960), and Mansfield (1961). The Bass model describes the diffusion process by the following differential equation:

$$\frac{dN(t)}{dt} = a\left[\bar{N} - N(t)\right] + \frac{b}{\bar{N}} \cdot N(t) \cdot \left[\bar{N} - N(t)\right] \quad \text{or} \tag{1}$$

$$\frac{dF(t)}{dt} = \left[a + bF(t)\right]\left[1 - F(t)\right] \tag{2}$$

where $N(t)$ is the cumulative number of adopters at time t, $\bar{N}$ is the ceiling or the population of potential adopters, a is the coefficient of innovation, b is the coefficient of imitation and $F(t) = N(t)/\bar{N}$, the fraction of the potential adopters who adopt the innovation by time t. The first term in (1) denotes the adoptions by innovators and the second term adoptions by imitators.

In recent years, some attempts have also been made to incorporate price (Robinson and Lakhani 1975; Bass 1980; Dolan and Jeuland 1981; Kalish 1983), advertising (Dodson and Muller 1978; Horsky and Simon 1983), promotion (Lilien, Rao and Kalish 1981), product inter-relationships (Peterson and Mahajan 1978), market size (Mahajan and Peterson 1978), repeat purchase (Dodson and Muller 1978; Lilien, Rao and Kalish 1981), and competition (Eliashberg and Jeuland 1982; Fershtman, Mahajan and Muller 1983; Mate 1982; Clarke and Dolan 1983) into this model. Furthermore, Bretschneider and Mahajan (1980) and Schmittlein and Mahajan (1982) have suggested feedback estimation and maximum likelihood estimation procedures, respectively, to estimate the model parameters.

In spite of the above extensions, there are still three conceptual limitations associated with these modeling efforts. First, they assume that the word-of-mouth effect remains constant over the entire time frame of the diffusion process, i.e., b is constant. Such an assumption is obviously inconsistent both with regard to theory and practice. There is no theoretical rationale for the word-of-mouth influence or the imitation effect to remain uniform over the entire penetration span. On the contrary, for most innovations, the imitation effect is more likely to increase, decrease or remain constant over time. Kotler (1971, p. 531), for example, has suggested that the coefficient of imitation "should decline through time, rather than stay constant, because the remaining potential adopters are less responsive to the product and associated communications." Although there appears to have been no behavioral studies of how influence changes with penetration, indirect evidence from several studies is available.

Coleman, Katz and Menzel (1966), in their study of the diffusion of a new drug, found that word-of-mouth eventually lost its importance. The reason for this was thought to be that, with the passage of time, and with increased use of the drug and observation of its effects and side-effects, the uncertainties associated with it were reduced. Consequently the need for physicians to discuss the drug likewise declined. Also a very large number of studies have found that early adopters are more socially integrated than late adopters (Rogers 1983). So it would be expected that the influence exerted by the early, more socially active, adopters would be greater than for the later, less socially active, adopters.

Hence, there is some evidence to suggest that influence may be higher initially. Certainly, the assumption that it remains uniform over the entire span of the diffusion process or that the effect of a contact between adopters and nonadopters is the same at 5% penetration as it is at 95% penetration is restrictive. In fact, Bundgaard-Neilsen (1976) has argued that late adopters may adopt faster than earlier ones, provided information concerning the innovation is available to all, because "late adopters are in a better position to assess the new technology than earlier ones." Consequently, the coefficient of imitation may be required to increase or decrease with time and the constant effect constraint of the current diffusion models may be unduly restrictive.

Second, the Bass model assumes that, for any innovation diffusion, the maximum rate of penetration, $(dN/dt)_{\max}$ or $(dF/dt)_{\max}$, cannot occur after an innovation has penetrated 50% of its potential market. The stage of the diffusion process at which the adoption rate is at a maximum is commonly referred to as the point of inflection. Setting the derivative of (2) with respect to F equal to zero and solving for F, one finds that for the Bass model the maximum penetration, F^*, occurs when $F^* = 1/2 - a/2b$. Hence, the Bass model does permit F^* to take on a range of values (less than or equal to 0.5) but since a is typically very small as compared to b (Bass 1969) this range is narrow. However, for a diffusion model to be flexible and to be able to accommodate various diffusion patterns, it is important that the model responds to the specific innovation diffusion pattern allowing the point of inflection to occur in either the earlier or later phase of development.

Third, the Bass model assumes that for any innovation, the diffusion curve is symmetric. That is, the diffusion pattern after the point of inflection (the stage of maximum adoption rate) is mirror image of the diffusion pattern before the point of inflection. Since $F^* = 1/2 - a/2b$,

$$\frac{dF}{dt} = \frac{a}{2} + \frac{b}{4} + \frac{a}{4b} - bK^2,$$

for $F = F^* + K$ as well as $F = F^* - K$, where K is a constant. Hence the Bass model presupposes that the diffusion curve is symmetric. Allowing the rate of adoption to be nonsymmetric is clearly of practical importance. Behaviorally, there is no reason to expect that adopters at $F^* - K$ and $F^* + K$ should respond in the same way. In fact, late adopters, possessing different characteristics from earlier adopters, may be expected to respond differently (Rogers 1983) and the diffusion models should allow for this.

Given the above limitations, the purpose of this paper is to present a nonuniform influence (NUI) diffusion model that overcomes these limitations. The approach is to suggest an extension of the Bass model which considers the coefficient of imitation as systematically varying over time. It will be shown that the proposed model allows: (i) the coefficient of imitation to increase, decrease or remain contant over time, and (ii) the diffusion curve to be symmetrical as well as nonsymmetrical with the point of inflection responding to the diffusion process.

The next two sections detail the NUI model development and its relative advantages over the current diffusion models in terms of the limitations discussed above. Data from five consumer durables are analyzed in §4 to illustrate the generality of the model. The paper concludes with the model limitations and directions for future research.

2. Nonuniform Influence Diffusion Model

In order to incorporate the time-varying nature of the word-of-mouth influence, the proposed model specifies the coefficient of imitation as systematically changing over time. More specifically, following the discussion in the preceding section, it is proposed that the coefficient of imitation be made a function of penetration. That is,

$$w(t) = b\left[\frac{N(t)}{\overline{N}}\right]^{\alpha} \tag{3}$$

where α is a constant and $w(t)$ represents the time-varying coefficient of imitation. Substitution of (3) into (1) yields:

$$\frac{dN(t)}{dt} = a\left[\overline{N} - N(t)\right] + b\left[\frac{N(t)}{\overline{N}}\right]^{\alpha}\frac{N(t)}{\overline{N}}\left[\overline{N} - N(t)\right]$$

$$= a\left[\overline{N} - N(t)\right] + b\left[\frac{N(t)}{\overline{N}}\right]^{\delta}\left[\overline{N} - N(t)\right] \quad \text{or} \tag{4}$$

$$\frac{dF(t)}{dt} = a\left[1 - F(t)\right] + bF(t)^{\delta}\left[1 - F(t)\right] \tag{5}$$

where $\delta = 1 + \alpha \geqslant 0$ and

$$w(t) = b\left[\frac{N(t)}{\overline{N}}\right]^{\delta - 1} = bF(t)^{\delta - 1}.$$

The Nonuniform Influence model can, therefore, be written as:[1]

$$\frac{dF(t)}{dt} = \left[a + bF(t)^{\delta}\right]\left[1 - F(t)\right]. \tag{6}$$

As compared to the Bass model, this model includes a nonuniform influence term to represent the word-of-mouth communication between adopters and nonadopters. If it is assumed that the diffusion takes place with uniform influence (i.e., the Bass model), then $\delta = 1$. However, the presence of a nonuniform influence effect in the diffusion process will be indicated by $\delta \neq 1$. The term δ will be referred to as the nonuniform influence factor.

Figure 1 illustrates the effect of the nonuniform influence factor on the diffusion curve. It is apparent from Figure 1 that values of the nonuniform influence factor, δ, between zero and one cause an acceleration of influence leading to an earlier and higher peak in the level of adoptions. Values of δ greater than one delay influence causing a later and lower peak.

With respect to the word-of-mouth communication, values of δ less than one cause the high initial coefficient of imitation to decrease with penetration. Values greater than one cause it to increase with penetration. This behavior can easily be confirmed by noting that

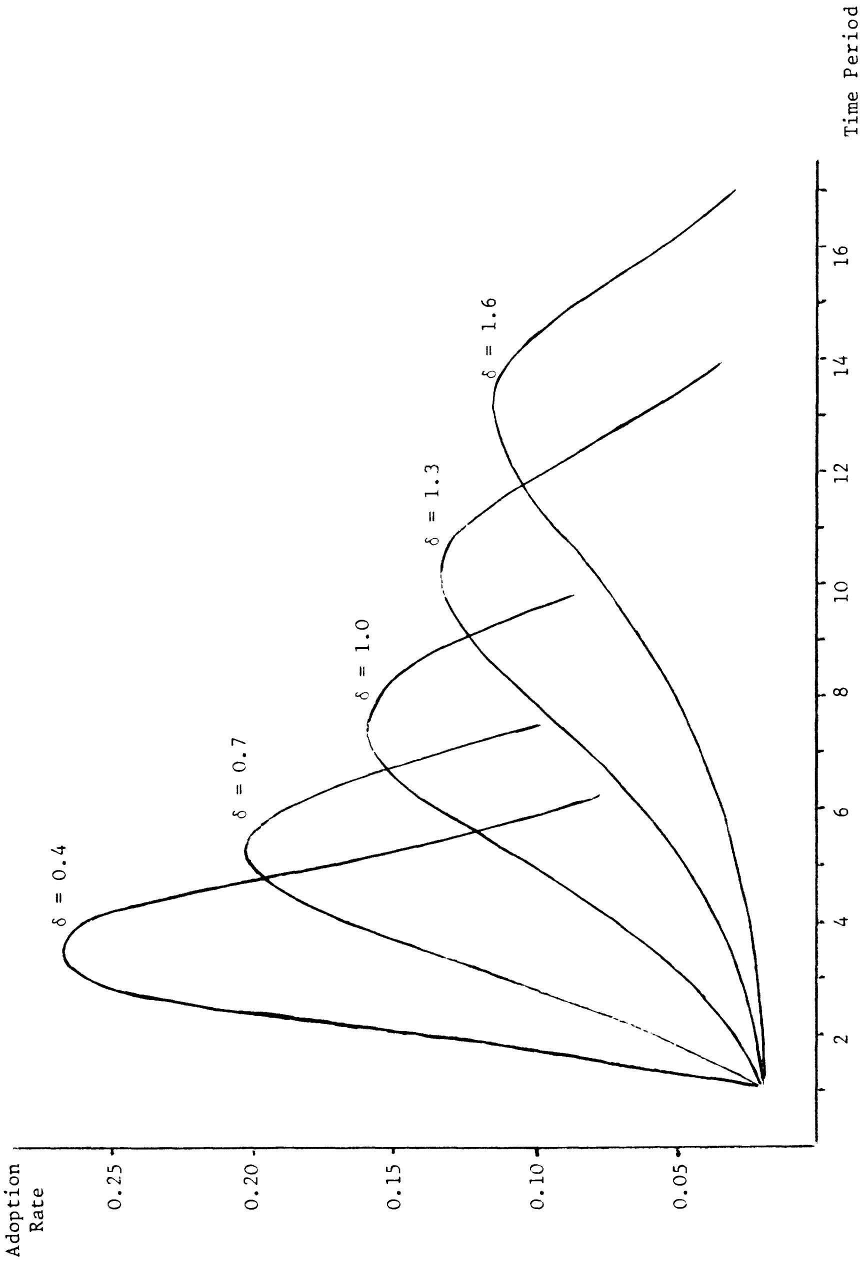
$$\frac{dw(t)}{dF} = b(\delta - 1)F(t)^{\delta - 2}$$

is less than zero for $0 \leqslant \delta < 1$, equal to zero for $\delta = 1$ and greater than zero for $\delta > 1$. Hence, since $dF/dt \geqslant 0$, the coefficient of imitation can increase over time (when $\delta > 1$), remain constant ($\delta = 1$) or decrease ($0 < \delta < 1$). Its

[1] The recognition that the diffusion model coefficients should be allowed to change over time has led to two main approaches. One is to introduce the factor causing the change into the diffusion model (e.g., Robinson and Lakhani 1975). The advantage of this approach is that the causal relationship is made explicit and can be measured and tested. However, many new product diffusions are characterized by the simultaneous interplay of several factors. It would, in such cases, be difficult, if not impossible, to statistically estimate these causal relationships, especially as there are usually few data points available for diffusion modelling.

It is in response to this type of situation that a second approach has been suggested in which no causal factors are introduced but the model parameters are allowed to vary freely in response to the data. Such an approach has been described by Bretschneider and Mahajan (1980). They use a feedback filter mechanism to revise the parameter estimates period by period. However, interpretation of the model parameters must take place entirely outside the model and, hence, may be difficult.

The method of estimating the time-varying coefficient of imitation used in the NUI model is clearly similar to the second approach. However, the coefficient of imitation is not free to move in any direction, period by period. The time-varying coefficient of imitation is constrained to belong to a family of curves described by $bF^{\delta - 1}$. This limits the response to the data. Freedom of movement has been lost for a gain in interpretability.



* a = 0.02, b = 0.6

FIGURE 1. Effect of δ on the Diffusion Curve.*

value at full market penetration is b. Hence, the time-varying nature of the coefficient of imitation is described by a family of curves given by $bF^{\delta-1}$.

The point of inflection for the NUI model can be obtained by differentiating equation (6) with respect to F, equating it to zero and solving for F^*. Differentiation of (6) and further simplification yields:

$$F(t)^{\delta-1}\left[\delta-(1+\delta)F(t)\right]=\frac{a}{b}. \tag{7}$$

There is no closed form solution of (7) for general δ and so in practice F^* would be obtained numerically. However, it is mathematically possible to obtain bounds on the values of F^*. As shown in the appendix, as $\delta \to \infty$, $F^* \to 1.0$ and as $\delta \to 0$, $F^* \to 0.0$, so that the point of inflection can vary from 0% to 100% penetration. In other words, unlike the Bass model, the NUI model allows the diffusion curve to attain its maximum rate of adoption at any stage of the diffusion process and is independent of the potential market captured by the innovation. Thus, the inflection point can happen early in the process or late depending on the diffusion process.

Because the NUI model has no closed form solution for F^*, it is not possible in general to illustrate analytically the nonsymmetrical property of the model. However it can be symmetrical, when $\delta = 1$ (i.e., the Bass model). That it can also be nonsymmetrical can be demonstrated by solving (7) numerically for F^*, and substituting $F = F^* \pm K$ in (6) and checking the nonsymmetry.

Hence, the NUI model provides an adoption rate curve which can be symmetrical or nonsymmetrical, with the point of inflection occurring at any stage of the diffusion process.

3. Comparison with Other Diffusion Models

In addition to the Bass model, a number of other models have been proposed to represent the diffusion imitation process. Most notable among these models are the Mansfield (1961) model (and its revised forms suggested by Blackman (1972) and Fisher and Pry (1971)), the Gompertz curve (Martino 1975), the Floyd (1962) model, the Sharif-Kabir (1976) model, and the extension of the Bass model suggested by Jeuland (1981). As stated below (dropping subscript t for convenience), except for the Jeuland model, all of these models formulate the diffusion process as a *pure* imitation process.[2]

[2]The original models proposed by Floyd (1962) and Sharif and Kabir (1976) are solutions of equations (10) and (11), respectively. The differential equation representations of these models are derived in Easingwood, Mahajan and Muller (1981). There does not appear to be a general solution for the NUI model. However, when δ is an integer and $1 \leqslant \delta \leqslant 6$, solutions for the NUI are available. When $a = 0$ in the NUI model, solution is obtainable when δ is an integer, and using a gauss hypergeometric function for any value of δ. The derivations of these solutions are given in the Wharton Marketing Department Working Paper No. 82-016.

Mansfield

$$\frac{dF}{dt} = bF(1 - F). \tag{8}$$

Gompertz Curve

$$\frac{dF}{dt} = bF\ln\left(\frac{1}{F}\right). \tag{9}$$

Floyd

$$\frac{dF}{dt} = bF(1 - F)^2. \tag{10}$$

Sharif-Kabir

$$\frac{dF}{dt} = \frac{bF(1 - F)^2}{\left[1 - F(1 - \sigma)\right]}, \tag{11}$$

where σ is a constant between zero and one.

Jeuland

$$\frac{dF}{dt} = (a + bF)(1 - F)^{1+\gamma} \tag{12}$$

where γ is a nonnegative constant.

The Sharif-Kabir model subsumes both the Mansfield model and the Floyd model. It yields the Floyd model when $\sigma = 1$ and the Mansfield model when $\sigma = 0$. The Jeuland model, on the other hand, subsumes the Bass model (when $\gamma = 0$), the Mansfield model (when $a = 0$ and $\gamma = 0$), and the Floyd model (when $a = 0$ and $\gamma = 1$). Table 1 provides a detailed comparison of the above five models, the Bass model and the NUI model on: (a) the nature of the coefficient of imitation, (b) location of point of inflection, and (c) symmetry/asymmetry. Some important observations on these models follow:

(a) *Coefficient of Imitation.* It is interesting to note that in addition to the NUI model, the Floyd, Sharif-Kabir and Jeuland models do allow the coefficient of imitation to systematically vary over time. In fact, these three models implicitly assume the following representations for the coefficient of

TABLE 1
Summary of Diffusion Model Characteristics

Model	Model Equation $dF/dt =$	Coefficient of Imitation	Location of point of inflection ($F^* =$)	Symmetry[1]
Mansfield	$bF(1-F)$	Constant	0.5	S
Gompertz	$bF\ln(1/F)$	Constant	0.37	NS
Floyd	$bF(1-F)^2$	Decreasing to 0.	0.33	NS
Sharif Kabir[2]	$\frac{bF(1-F)^2}{1-F(1-\sigma)}$	Constant or decreasing to 0	0.33 thru 0.5	S or NS
Bass	$(a+bF)(1-F)$	Constant	0.0 thru 0.5	S
Jeuland[3]	$(a+bF)(1-F)^{1+\gamma}$	Constant or decreasing to 0	0.0 thru 0.5	S or NS
NUI	$(a+bF^{\delta})(1-F)$	Increasing, decreasing or constant	0.0 thru 1.0	S or NS

[1]Symmetrical is denoted by S, nonsymmetrical by NS.
[2]$0 \leqslant \sigma \leqslant 1$.
[3]$0 \leqslant \gamma$.

imitation:

Floyd

$$w(t) = b(1-F), \tag{13}$$

Sharif-Kabir

$$w(t) = \frac{b(1-F)}{[1-F(1-\sigma)]}, \tag{14}$$

Jeuland

$$w(t) = b(1-F)^{\gamma}. \tag{15}$$

However, in all of the three models, the coefficient of imitation can only decrease with time. For the Sharif-Kabir model, differentiation of (14) with respect to F gives

$$\frac{dw(t)}{dF} = \frac{-b\sigma}{[1-F(1-\sigma)]^2}.$$

Since b is positive and $0 \leqslant \sigma \leqslant 1$, $dw(t)/dF$ is negative and hence $w(t)$ must decrease with time. Similarly, for the Jeuland model, since

$$\frac{dw(t)}{dF} = -b\gamma(1-F)^{\gamma-1},$$

the coefficient of imitation must decrease with time. In fact, for all the three models it becomes zero when F reaches unity. The NUI model is the only model which allows the coefficient of imitation to decrease, increase or remain constant with penetration.

(b) *Point of Inflection.* Other than the NUI model, none of the models allow inflection points beyond 50% penetration. However, in the Bass and Jeuland models, the inflection point can occur anywhere on or before the 50% penetration. The range, however, for the Sharif-Kabir model is restricted between $33\frac{1}{3}\%$ and 50% penetration. Differentiation of the Sharif-Kabir model, equation (11), with respect to F and equating it to zero yields:

$$F^2(2-2\sigma) - 3F + 1 = 0. \tag{16}$$

Hence, when $\sigma = 1$, equation (16) gives $F^* = 1/3$, the inflection point for the Floyd model, and when $\sigma = 0$, it gives $F^* = 1/2$, the inflection point for the Mansfield model. Similarly, it can be easily shown that the point of inflection for the Jeuland model is given by:

$$F^* = \frac{1}{2+\gamma}\left(1 - (1+\gamma)\frac{a}{b}\right). \tag{17}$$

Substitution of $\gamma = 0$ in (17) yields the inflection point for the Bass model, and hence the upper limit for the inflection point is 50%. From (17), it is evident that $F^* = 0$ when $b = a(1+\gamma)$.

(c) *Symmetry.* In addition to the NUI model, only the Sharif-Kabir and Jeuland models allow the diffusion curve to be symmetrical as well as nonsymmetrical. Both the Bass and Mansfield models provide symmetrical diffusion curves. The Gompertz and Floyd models require diffusion curves to be nonsymmetric.

The above comparisons clearly illustrate that as compared to all of the notable current diffusion models, the NUI model is the only model which allows the coefficient of imitation to decrease, increase or remain constant over time. It allows the diffusion curve to be symmetrical as well as nonsymmetrical with the point of inflection responding to the diffusion process.[3]

[3]Note if the coefficient of innovation, a, is equal to zero, (6) gives the nonuniform influence model for the Mansfield model. The special case of the NUI model when $a = 0$ has been termed as the Nonsymmetric Responding Logistic (NSRL) model and is discussed in detail in Easingwood, Mahajan and Muller (1981).

It has been shown in the paper that the Gompertz curve also possesses a fixed point of inflection at 37% penetration and is nonsymmetric. The flexible point of inflection in the Gompertz curve can also be obtained by using the proposed systematic time-varying coefficient of

At this stage it is important to note two additional distributions which have been used to model diffusion patterns, namely, the normal distribution (Stapleton 1976) and the Weibull distribution (Pessemier 1977; Sharif and Islam 1980). The cumulative normal distribution yields a S-shaped curve with a fixed point of inflection occurring at $F^* = 0.5$. Hence the utilization of a normal distribution to generate diffusion patterns is also restrictive. Pessemier (1977) and Sharif and Islam (1980) have suggested the use of the Weibull distribution as a general model to forecast innovation diffusion. This curve provides nonsymmetry with a point of inflection that responds to the penetration patterns. However, the model, although offering the desirable diffusion curve properties, ignores the underlying theory behind the diffusion process. Furthermore, the behavioral and managerial interpretations of the parameters characterizing the model are not clear.

4. Illustrations

In order to illustrate the generality of the NUI model, sales data for five consumer durables were analyzed. The durables included were black and white televisions, color televisions, clothes dryers, room air conditioners and dishwashers. The time series were restricted to the early years of sales growth to help avoid replacement or repeat purchases, and the data were collected from the *Statistical Abstracts of the United States*. Furthermore, in order to assess the relative performance of the NUI model, these durables were also analyzed by using the Bass and Sharif-Kabir models.[4] The associated parameters of the models were computed by using a nonlinear programming algorithm (Wismer and Chattergy 1978). In each case the deterministic model that best fitted the data was obtained. That is, the algorithm searched for the parameter values that most closely approximated the actual diffusion curve. The parameter estimates, the fit statistics and the derived points of inflection for the three models are reported in Table 2. Actual and NUI fitted values are illustrated in Figure 2. The time-varying coefficient of imitation for the NUI model is shown plotted against time in Figure 3. The following comments on these analyses are warranted:

· The fitted values of the nonuniform influence factor, δ, of the NUI model were less than 1.0 for four of the five products, indicating high initial influence that decreases with penetration. The products are black and white televisions ($\delta = 0.3$), color televisions ($\delta = 0.6$), clothes dryers ($\delta = 0.7$) and room air

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imitation, i.e.,

$$\frac{dF}{dt} = bF^{\delta}\ln\left(\frac{1}{F}\right).$$

It can be easily shown that for this curve the point of inflection is $F^* = e^{-1/\delta}$. The curve, however, must always be nonsymmetric.

[4]Both the NUI and the Jeuland models suggest inclusion of an additional parameter into the Bass model to attain further flexibility. However, since the NUI model offers the most flexibility, only the NUI model is compared with the Bass and the Sharif-Kabir models.

TABLE 2

Parameter Estimates, Fit Statistics and Points of Inflection for Consumer Durables

	Products*														
	Black & White Television (1947–53)			Color Televisions (1963–70)			Clothes Dryers (1949–61)			Room Air Conditioners (1949–61)			Dishwashers (1949–61)		
	Bass	Sharif-Kabir	NUI	Bass	Sharif-Kabir	NUI	Bass	Sharif-Kabir	NUI	Bass	Sharif-Kabir	NUI	Bass	Sharif-Kabir	NUI
a	0.03297	—	0.000,021	0.03297	—	0.010,610	0.01856	—	0.008,794	0.01386	—	0.000,168	0.01164	—	0.018,380
b	0.6486	1.67260	0.2805	0.6352	1.2159	0.4121	0.3296	0.7857	0.2494	0.3882	0.8025	0.2013	0.2134	0.4097	0.3717
$\overline{N}$	39,928	32,260	48,096	36,845	39,253	41,678	15,914	14,402	17,233	17,772	17,359	22,389	10,077	9,468	9,987
σ	—	1.0	—	—	1.0	—	—	1.0	—	—	1.0	—	—	1.0	—
δ	—	—	0.3000	—	—	0.6000	—	—	0.7179	—	—	0.4954	—	—	1.5437
Mean Absolute Deviation	1101.1	1735.1	836.5	379.5	732.2	228.7	106.0	286.6	92.0	156.1	281.7	105.6	41.3	55.8	31.5
Mean Squared Error	1770.9	5424.9	1024.6	196.3	814.8	75.4	18.8	95.8	12.9	31.1	113.6	14.9	2.3	6.0	1.5
Explained Variance (adjusted %)	64.6	7.2	72.8	93.0	74.5	96.7	88.7	47.0	91.5	90.4	68.0	94.9	86.9	70.5	91.5
Point of Inflection	0.47	0.33	0.23	0.47	0.33	0.36	0.47	0.33	0.40	0.48	0.33	0.33	0.47	0.33	0.58

*Estimates of $\overline{N}$ are in thousands. Mean squared error is in thousands.

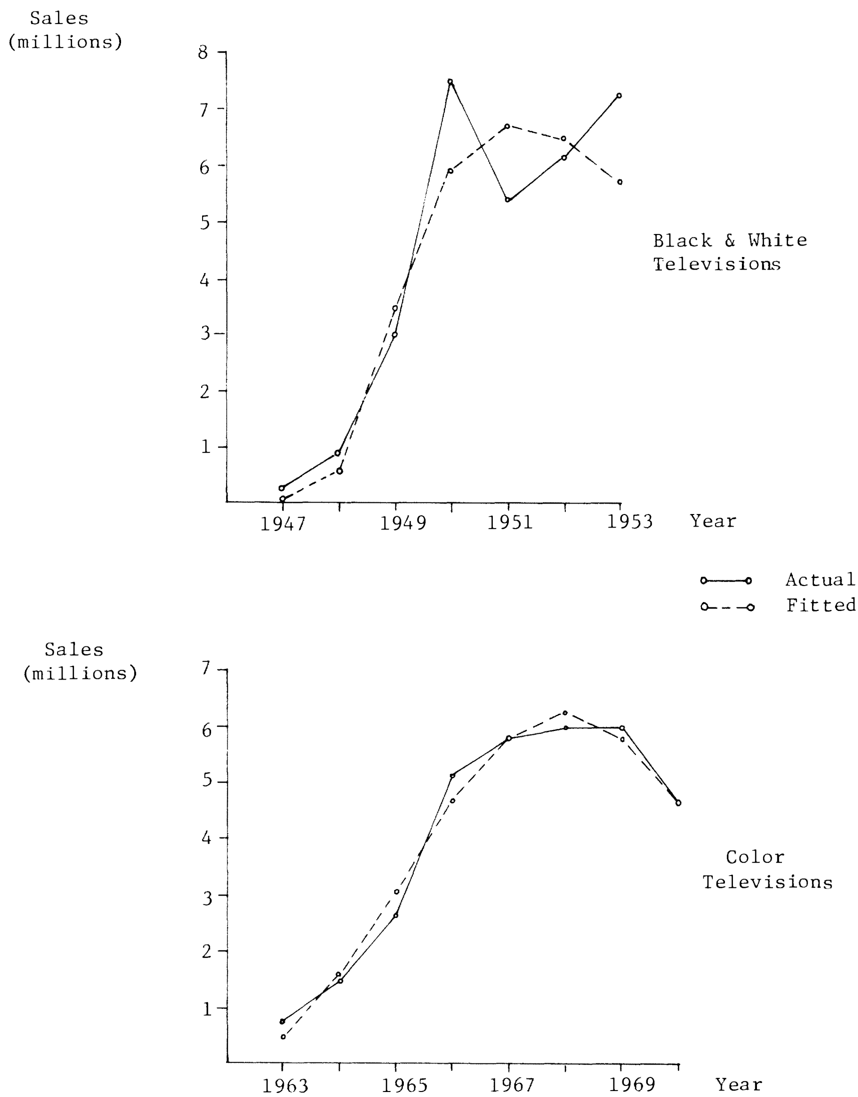


FIGURE 2. Actual and NUI Fitted Sales.

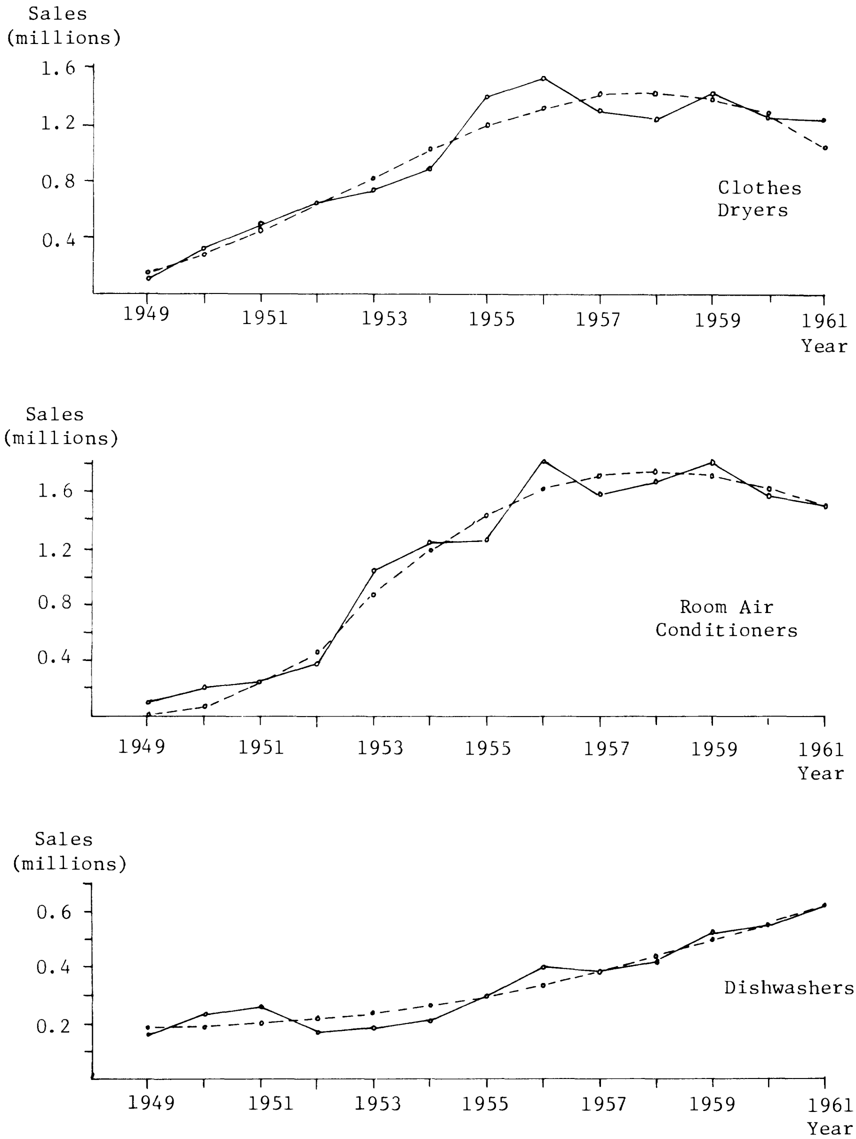


FIGURE 2. (*continued*) Actual and NUI Fitted Sales.

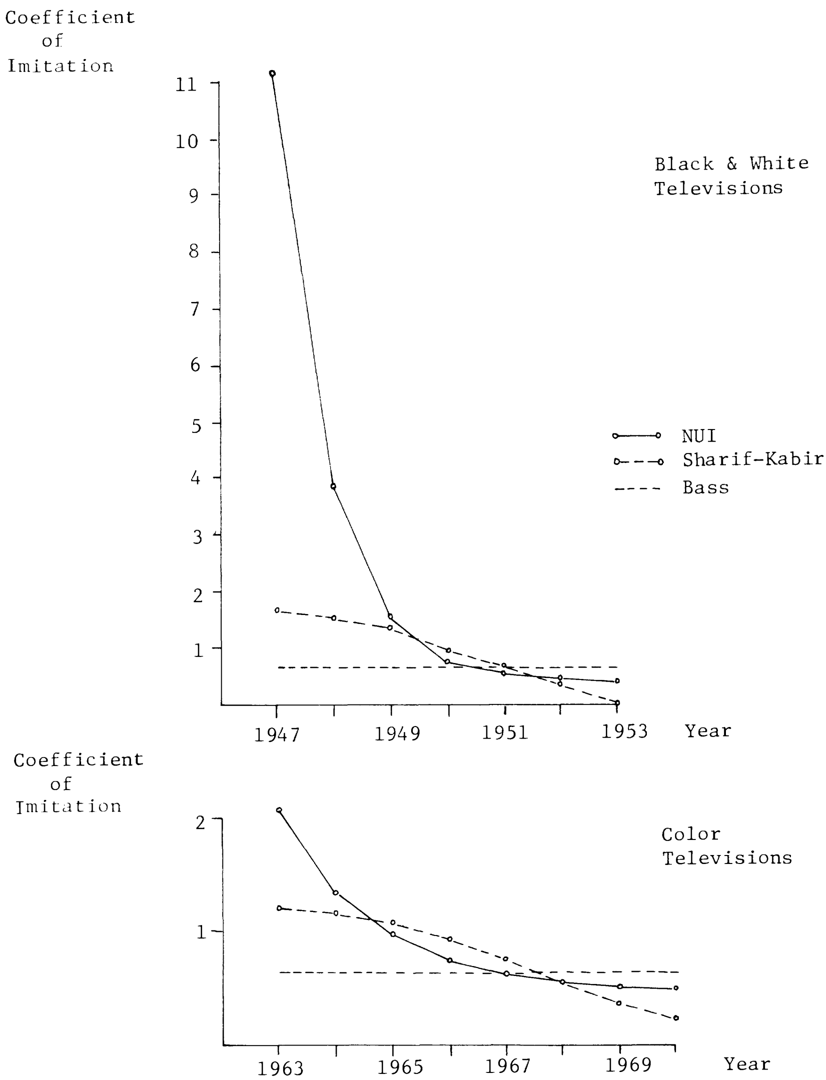


FIGURE 3. Comparison of Coefficient of Imitation.

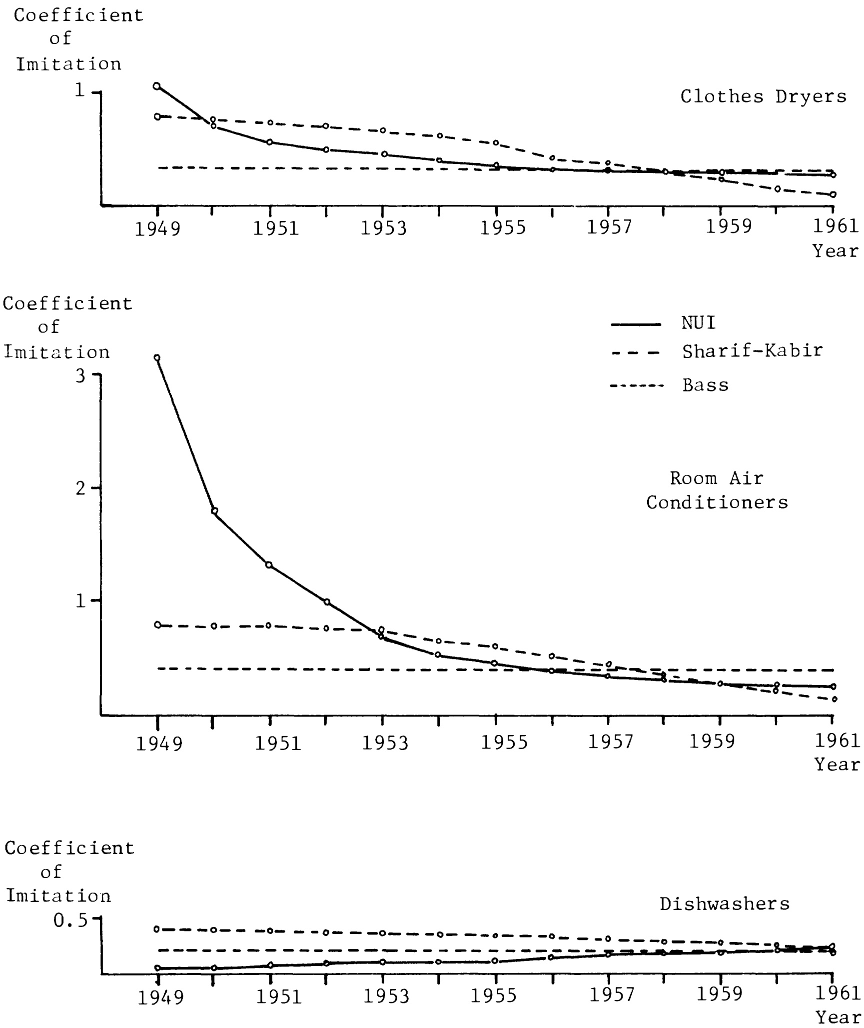


FIGURE 3. (*continued*) Comparison of Coefficient of Imitation.

TABLE 3

Two and Four Steps Ahead Forecasts

		Forecast Period	Mean Absolute Error*		Mean Squared Error+	
			Bass	NUI	Bass	NUI
I	*Two Steps Ahead*					
	Black & White Televisions	1952–53	6,656	923	44,609	1,178
	Color Televisions	1969–70	3,353	254	11,572	78
II	*Four Steps Ahead*					
	Clothes Dryers	1958–61	703	95	601	15
	Room Air Conditioners	1958–61	1,070	56	1,267	4
	Dishwashers	1958–61	76	42	7	2

*Measured in 000s.

+ Mesured in 000,000,000s.

conditioners ($\delta = 0.5$). The only product with a value of delta greater than unity, indicating increasing influence, was dishwashers ($\delta = 1.5$). The value of σ in the Sharif-Kabir model was 1.0 for all five products, indicating that the Sharif-Kabir model reduces to the Floyd model in each case (see Table 2).

· Table 2 indicates that the NUI model provides a better fit than the other two models for each of the five consumer durables. The average percentage of variance explained (adjusted) was 85% for the Bass model, 53% for the Sharif-Kabir model and 90% for the NUI model. The NUI model produces a sizeable reduction in the mean absolute error and in the mean squared error. The mean absolute errors for the Bass and Sharif-Kabir models are, on average, 38% and 157% higher, respectively, than for the NUI model. The mean squared errors for the Bass and Sharif-Kabir models are, on average, 88% and 603% higher, respectively, than for the NUI model.

· The NUI estimates of the coefficient of imitation vary considerably over the time periods analyzed. Initial period estimate of the coefficient of imitation is 28.5 times final period estimate for black and white televisions, 4.5 times final period estimate for color televisions, 3.9 times final period estimate for clothes dryers and 62.4 times final period estimate for room air conditioners.[5] For dishwashers, the only product with an increasing coefficient of imitation, final period value is 6.0 times initial period value (see Figure 3).

· The fitted values of the points of inflection are between 0.23 and 0.58 for the NUI model but between only 0.47 and 0.48 for the Bass model and are equal to 0.33 in all cases for the Sharif-Kabir model (see Table 2).

The predictive ability of the Bass and NUI models was compared by removing the last two data points of the shorter series (black and white televisions and color televisions) and the last four data points of the longer series (clothes dryers, room air conditioners and dishwashers) and fitting the models to the truncated data. Then forecasts were produced for all the missing years by extrapolating the fitted models. The results are reported in Table 3. The average mean squared error for the missing years is higher for the Bass model compared to the NUI model; 37.9 times higher for black and white televisions, 148.4 times higher for color televisions, 40.1 times higher for clothes dryers, 316.8 times higher for room air conditioners and 3.5 times higher for dishwashers.

In a recent paper Schmittlein and Mahajan (1982) have suggested that the fit and predictive reliability of the Bass model can be improved by using maximum likelihood estimation procedures. Given the superiority of the maximum likelihood version of the Bass model proposed by Schmittlein and Mahajan over the least squares procedure, Table 4 compares the relative performance of the NUI model and the MLE-Bass model for the consumer durables reported by Schmittlein and Mahajan. The fit and forecast statistics for the NUI model are abstracted from Table 2 and the corresponding MLE-Bass statistics are taken from Tables 2 and 3 of Schmittlein and

[5]Some of these ratios are very high because initial period market share is very low leading to very high initial period values.

TABLE 4

*Comparison of the NUI Model and the MLE-Bass Model**

	Fit Statistics				Forecast Performance Statistics			
	Mean Absolute Deviation		Mean Squared Error		Mean Absolute Deviation		Mean Squared Error	
Product	NUI	MLE	NUI	MLE	NUI	MLE	NUI	MLE
Color Televisions	228.7	232.0	75,400	121,496	254	2559.93	78,000	8,529,629
Clothes Dryers	92.0	95.6	12,900	16,106	95	284.97	15,000	135,904
Room Air Conditioners	105.6	136.6	14,900	28,759	56	426.47	4,000	245,072
Diswashers	31.5	33.1	1,500	1,688	42	NA	2,000	NA

*The MLE-Bass statistics are abstracted from Schmittlein and Mahajan (1982, Tables 2 and 3). The forecast performance statistics for the MLE-Bass model are for one-step-ahead forecasts and these statistics are not available for dishwashers.

Mahajan. Table 4 clearly indicates the consistently better relative performance of the NUI model over the MLE-Bass model. For example, the average mean squared errors for the MLE-Bass fit statistics are higher than for the NUI model in all four cases: 61.1% higher for color televisions, 24.9% higher for clothes dryers, 93.0% higher for room air conditioners and 12.5% higher for dishwashers.

Thus, the above analyses suggest that the NUI model fits the actual data well and by means of the flexibility introduced by the nonuniform influence factor allows the model to respond to the diffusion process.

5. Discussion and Conclusions

Since the original work by Bass (1969), a number of models have been proposed to represent the time-dependent aspects of the diffusion process. In recent years, however, the viability of diffusion models to forecast the new product/technology growth has been re-examined (Bernhardt and MacKenzie 1972; Heeler and Hustad 1980; Sharif and Islam 1980; Schmittlein and Mahajan 1982). Bernhardt and MacKenzie (1972), for example, have stated that in some cases the simple diffusion models work well and in other cases the results are not as good. They suggest that the success of diffusion models has been due to "judicious choice of situation, population, innovation and time frame for evaluating the data." Heeler and Hustad (1980) report examples of new product diffusion in an international setting where the Bass model does not perform well. In this paper, we re-examined the basic structure of innovation models in terms of three assumptions related to the word-of-mouth effect, point of inflection and symmetry. As summarized in Table 1, the existing models offer no or limited flexibility in accommodating these assumptions. This limits their ability to accommodate different diffusion patterns and might explain why these diffusion models work in some cases and do not provide good results in others.

This paper has suggested a simple diffusion model termed as Nonuniform Influence (NUI) diffusion model. By allowing the coefficient of imitation to systematically vary over time, the model can accommodate different diffusion patterns. It allows the diffusion curve to be symmetrical as well as nonsymmetrical, with the point of inflection responding to the diffusion process.

Given that the flexibility offered by the NUI model is determined by the nonuniform influence factor δ, work is currently underway to develop a taxonomy of diffusion patterns based on δ and the coefficient of imitation. Lawrence and Lawton (1981), for example, have found that the coefficient of imitation is lower for consumer durables than for industrial products. While this result, if confirmed, has potential use in forecasting product life cycles prior to the product development and introduction, it represents a limited analysis of the diffusion patterns. A detailed taxonomy based on the coefficient of imitation and the nonuniform influence factor would provide a better understanding of the dynamics of diffusion patterns and hopefully would lead to a viable procedure to develop pre-launch forecasts.

In the presentation and applications of the NUI model, we have assumed that for a particular innovation the word-of-mouth effect can either increase, decrease or remain constant over the diffusion span. However, it is highly likely that for certain innovations the word-of-mouth effect may have a pattern where the effect increases in certain periods and decreases or remains constant in other periods. The proposed model is capable of capturing such a behavior. However, this would require usage of time-varying estimation procedures such as feedback approaches (Mahajan, Bretschneider and Bradford 1980) to estimate the NUI model parameters. Work is currently underway to explore the usage and applications of such approaches to represent such behaviors.

Finally, given that the proposed model possesses the desirable basic properties, further work is also needed to incorporate the marketing programming variables, dynamic market potential, negative-word-of-mouth and competition into this model.

Appendix

The point of inflection for the NUI model is given by differentiating (6), equating to zero and solving for F^*. However, as mentioned in §2, there is no closed form solution for F^* for general δ and so in practice F^* will be obtained numerically. However, it is possible to obtain bounds on the values of F^*. The objective of this appendix is to show that as $\delta \to 0$, $F^* \to 0.0$ and as $\delta \to \infty$, $F^* \to 1.0$, so that the point of inflection for the NUI model can vary from 0% to 100% penetration.

I. $\delta = 0$, $F^* \to 0.0$

Differentation of the NUI model, equation (6), with respect to F and equating it to zero yields

$$F^{\delta-1}\big[\delta - (1+\delta)F\big] = \frac{a}{b} \quad \text{or} \tag{A-1}$$

$$\big[\delta - (1+\delta)F\big] = F^{1-\delta} \cdot \frac{a}{b}\,. \tag{A-2}$$

As $\delta \to 0$, LHS (of equation (A-2)) $\to -F$ while RHS $\to a/b \cdot F$. The only way these two limits can be equal $(a, b \geqslant 0)$ is to let $F \to 0$. Hence, as $\delta \to 0$, $F^* \to 0.0$.

II. $\delta \to \infty$, $F^* \to 1.0$

In order to establish this relationship it is first necessary to show the point of inflection, F^*, for the NUI model is given by:

$$\frac{\delta - 1}{\delta + 1} < F^* < 1. \tag{A-3}$$

Then, given (A-3) it can be easily shown that

$$\lim_{\delta\to\infty}\left(\frac{\delta-1}{\delta+1}\right)=1.$$

Hence, as $\delta\to\infty$, $F^*\to 1$. That is,

$$\lim_{\delta\to\infty}\left(\frac{\delta-1}{\delta+1}\right)=\lim_{\delta\to\infty}\left(\frac{\delta}{\delta+1}-\frac{1}{\delta+1}\right)$$

$$=\lim\frac{1}{1+1/\delta}-\lim\left(\frac{1}{\delta+1}\right)=1-0=1.$$

Equation (A-3) can be established by examining the second derivative of the NUI model. Only the outline of the proof is given here and details can be found in the Wharton Marketing Department Working Paper No. 82-016.

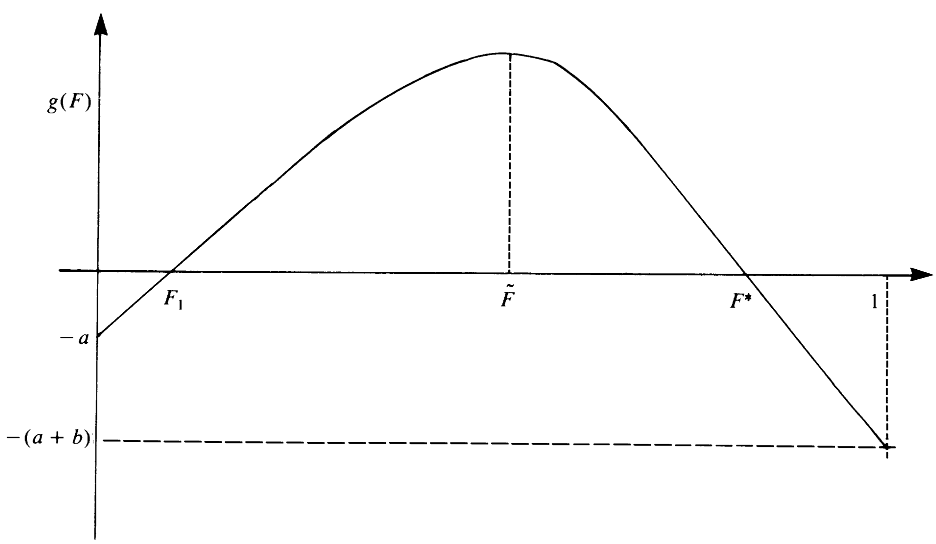
Let $f(F)=(a+bF^{\delta})(1-F)$. Let $\delta>1$. Let

$$g(F)=df/dF=-(1+\delta)bF^{\delta}+b\delta F^{\delta-1}-a.$$

Hence, $dg/dF=-\delta(1+\delta)bF^{\delta-1}+b\delta(\delta-1)F^{\delta-2}$. Given the second derivative, it is straightforward to check that $g(F)$ has a unique maximum at $\tilde{F}=(\delta-1)/(\delta+1)$. If $a<b((\delta-1)/(\delta+1))^{\delta-1}$ then $g(F)>0$ and g has the shape as depicted below where

$$0<F_1<\frac{\delta-1}{\delta+1}<F^*<1.$$

$F(t)$ thus has at most two inflection points at F_1 and at F^*.



However if $F_0 > 0$ and is large enough so that $g(F_0) > 0$ (i.e.,

$$\left[b\delta F_0^{\delta-1} - (1+\delta)bF_0^{\delta} \right] > a)$$

then (and only then) g will have only one vanishing point at F^*. In this case it is clear that $g(\tilde{F})$ is indeed positive.

With a, b and δ being positive, the condition that $g(F_0) > 0$ is clearly both necessary and sufficient for $F(t)$ to have a unique inflection point at F^*. In case that $\delta < 1$, then $dg/dF < 0$ for all F. Since $g(1) < 0$, F has a unique inflection point if and only if $g(F_0) > 0$.

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