MARKET SHARE PIONEERING ADVANTAGE: A THEORETICAL APPROACH*

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In this paper we present an industry in which a pioneer has entered, accumulated capital in the form of goodwill, and in his monopoly period has also reduced his cost of production as a result of some form of learning by doing. At some later date a newcomer enters. His production cost is higher than that of the monopolist at that date. However due to diffusion of information, the costs of the two firms equate at some future date.

Once the new firm enters the market a duopolistic game begins in which the firms choose prices and investment rates. Analyzing this game we discover the conditions under which the final market shares no longer depend on the order of entry, the initial cost advantage, the length of the monopoly period, or the length of time it took the newcomer to overcome the pioneer's cost advantage.

We analyze the speed and pattern of convergence to the final market shares and the capital path of the pioneer in his monopoly period, depending on his beliefs concerning the possibility of entry.

(MARKETING—COMPETITIVE STRATEGY, NEW PRODUCTS, PRICING AND AD-VERTISING; DIFFERENTIAL GAMES)

1. Introduction

The success of a firm depends to a large degree on its ability to produce new, innovative products or services. If the new product is so innovative as to start a whole new market and industry, the firm that has introduced it is usually referred to in the literature as a pioneer. It is risky and expensive to be a pioneer. The rewards, though, might be an advantage that translates to larger market share and profits.

In this paper we present a relatively simple model of an industry in which the firm that enters first, i.e., the pioneer, accumulates goodwill or another form of capital for some time until a newcomer enters, then starts a game in which both compete on pricing and advertising.

Our main interest is the effect of the order of entry on market share position. We define the advantage of a pioneer in terms of final market share of the same firm i in two situations; one in which it enters first and its competitor second, and one in which the entry times are reversed.

Note that market share pioneering advantage refers only to the lasting market-share advantage and not to the extra discounted profits a pioneering firm might get. We do, however, investigate the pattern and speed of convergence to these final market shares as well.

Several authors, including Robinson and Fornell (1985), have argued that it is not only the order of entry *per se* that affects market shares but that the order of entry gives the pioneer an advantage such as cost of production, cost of advertising, brand loyalty, quality, and the like. These advantages translate into market-share differential. Thus we can say that the main concern of this paper is the long-term effect of any such advantage.

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^{*} Accepted by Jehoshua Eliashberg; received January 10, 1987. This paper has been with the authors 17 months for 4 revisions.

Clearly, as long as the cost advantage (or advantage in terms of quality, brand loyalty and the like) remains, we expect the advantage in terms of the market share to remain as well. The interest is when at some future date, the newcomer has equated the cost and elasticity parameters. Will the newcomer then slowly erase the market-share differential or will it be permanent? Does it depend in any way on the length of time during which the pioneer commands an advantage over the newcomer, or the length of time during which the pioneer has been the sole producer in the industry?

The general answer we give in this paper indicates the transitory effect of such advantages. Consider, for example, the following scenario. The pioneer entered at time T_1 . In his monopoly period he accumulates capital in the form of goodwill and, through learning by doing or other reasons, he also reduces his production cost. At some later date T_2 , a newcomer enters. Since diffusion of technological innovation is not instantaneous, the newcomer's production costs are larger than those of the monopolist at this date T_2 . They might be lower than the monopolist's costs at T_1 , because some diffusion of information has occurred in the interim period. In the duopoly that now evolves, initial goodwill levels and initial market shares are of course very much lopsided to the advantage of the pioneer.

Because of later diffusion of information and because of some technological innovation (possibly by an outsider), at some later date T^* , costs of production of the two firms may equate at a common cost c.

We now watch the industry evolve and become mature. When "the dust has settled" we measure the final market shares. What we find in this work are the conditions under which these final market shares do not depend on the order of entry, or on the initial cost advantage, or on the length of the monopoly period $T_2 - T_1$, or on the length of the period that it took the newcomer to overcome the pioneer's cost advantage $T^* - T_2$.

In the monopoly period we analyze the effect of anticipation of entry on the incumbent's decisions. What we find is that the monopolist who does not anticipate entry overcapitalizes as compared to the monopolist who correctly predicts the time of entry.

The model we use is a differential game model that has been used extensively in the recent marketing literature on competitive behavior for new products. (For a recent review see Eliashberg and Chatterjee 1985 or Dolan, Jeuland and Muller 1985.) In particular, the model complements the efforts of Deal (1979), Teng and Thompson (1983) Thompson and Teng (1984), Rao (1982), Eliashberg and Jeuland (1986) and Dockner and Jorgensen (1988) in examining the diffusion of a new product in the realistic setting of market competition. The main difference between these works and ours is in the objective of the papers. Eliashberg and Jeuland concentrate mainly on the effects of entry on pricing strategies. Teng and Thompson examine the effects of competition on advertising policies when the oligopolists "learn by doing." Rao investigates the problem of investment on goodwill in a model which is similar to ours and looks for conditions which will guarantee local stability of the steady state, while this work investigates the importance of being a pioneer in a given industry.

2. Model Formulation

Consider a situation in which the firms enter the market consecutively, so that the first period in which only one firm is in the market is a monopoly period, while in the second period, when the second firm has entered as well, we have a duopolistic market. For convenience we denote by zero the time of the beginning of the duopoly game, i.e., if firm one (the pioneer) entered at time T_1 and the second firm entered in time T_2 , we employ the time translation $t - T_2$ to arrive at zero being the starting time of the duopoly game. Let $T_1 < T_2$ be two entry times. Let $MS_i(T_1, T_2, t)$ be the market share of firm *i*

at time t, when firm i entered at time T_1 and its competitor entered at time T_2 , and let $MS_i^*(T_1, T_2)$ be the limit if there is one to which the market shares converge over time.

DEFINITION.¹ In an industry there is *market share pioneering advantage*, if for some $T_1 < T_2$, and firm *i*, the following inequality holds:

$$MS_i^*(T_1, T_2) > MS_i^*(T_2, T_1).$$

The goodwill levels of the two firms at time t are denoted by x(t) and y(t), respectively. For simplicity, we model the change over time of the goodwills to behave according to the well-known Nerlove-Arrow (1962) goodwill accumulation equation.

$$\dot{x}(t) = u_1(t) - \delta_1 x(t); \qquad x(0) = x_0,$$
(1.1)

$$\dot{y}(t) = u_2(t) - \delta_2 y(t); \qquad y(0) = y_0,$$
 (1.2)

where δ_i is the goodwill depreciation parameter of firm *i*, a dot above a variable represents differentiation with respect to time and u_i is the advertising effort of firm *i*. The cost of the effects of u_i , which is assumed to be in a compact set $[0, \bar{u}_i]$, is given by $C_i(u_i)$ for some convex cost function *C*. For example, the cost $C_i(u_i)$ which is convex and satisfies the lim $C_i \rightarrow \infty$ as $u_i \rightarrow \bar{u}_i$ will induce a control function as desired.

A problem such as ours with convex cost and linear effectiveness function is structurally equivalent to a problem with linear cost and concave effectiveness function. For example an exponential cost function will precisely correspond to the logarithmic advertising effectiveness function used in Horsky and Simon (1983). In our model the advertising effectiveness function is C_i^{-1} , i.e., the inverse of $C_i(u_i)$.

We assume in this paper that firms' sales are functions of the respective goodwill levels as well as the prices the firms charge. We follow Parsons and Bass (1971) by assuming a log-linear functional relationship between goodwill levels and sales. A log-linear function relating capital to production or sales has been used extensively in economics since the early 1900s (see for example Thompson 1981), and the first to use it in an oligopoly setting were Parsons and Bass. The advantage of such a function is first its flexibility, and second its ease of use for empirical estimation. Specifically we assume that sales of firm *i*, denoted by s_i , are given by:

$$s_1 = x^{\alpha_1} y^{-\beta_1} b_1(p_1, p_2), \qquad (2.1)$$

$$s_2 = y^{\alpha_2} x^{-\beta_2} b_2(p_1, p_2). \tag{2.2}$$

Thus the "coupling" of one firm to the other, i.e., the way one oligopolist effects his rival is done via: (a) the effect of pricing (the short-term variable) on demand and (b) the effect of goodwill (the long-term variable) on demand. The goodwill accumulation equation is thus left "uncoupled." In order to ensure boundedness of the sales function we rescale the variables x and y so that their lower bound is one instead of zero.²

¹ We restrict the definition to cases in which the firm's market shares converge to some steady state level of MS_i^* . In cases in which there is no such convergence, the definition given does not apply. There might then be several alternative definitions. For example, one can require that at the lowest level the inequality holds, i.e.,

$$\liminf_{t\to\infty} MS_i(T_1, T_2, t) > \liminf_{t\to\infty} MS_i(T_2, T_1, t).$$

Another possibility is to strengthen the requirement to one in which the inequality holds from a certain time onwards, i.e., that there exists a time T, such that for all t > T, the following inequality holds:

$$MS_i(T_1T_2, t) > MS_i(T_2, T_1, t).$$

² Alternatively we could use a somewhat different function such as $s_1 = x^{\alpha}(1 + y)^{-\beta}b_1(p_1, p_2)$. In addition, our propositions and the proof of convergence of §5 will work with a multiplicative separable function, i.e., $dx/dt = g(x)h(y)b(p_1, p_2)$. We lose, however, the parsimonious representation of the conditions for convergence, equation (10).

The parameters α and β have the following intuitive interpretation as elasticities: a one percent increase in the firm's own goodwill will increase its sales by α_1 percent. Similarly a one percent increase in the rival's goodwill will decrease the firm's sales by β_1 percent. The restrictions on the parameters α and β are as follows. First, the concavity needed for sufficiency conditions requires that α be bounded between zero and one. Second, it is reasonable to require the industry's sales to increase when goodwill levels of both firms increase; this results in the requirement that α_i is larger than β_i .

We thus conclude that the following inequality holds:

$$0 < \beta_i < \alpha_i < 1. \tag{3}$$

As will be discussed later, stability requirements will further constrain the parameters so that $\alpha_i + \beta_i < 1$.

In order to see the implications of this condition assume a symmetric situation with respect to prices and the parameters α_i and β_i . Substituting from (2) we have:

$$s_1/(s_1 + s_2) = x^{\alpha+\beta}/(x^{\alpha+\beta} + y^{\alpha+\beta}).$$
 (4)

Thus if $\alpha + \beta = 1$, market shares are exactly equal to goodwill shares. This is similar in spirit to the model of Horsky (1977), who assumed that the firm's market shares depend only on the respective goodwill levels. Specifically in his model at the steady state the following relationship holds:

$$s_1/(s_1 + s_2) = k_1 x/(k_1 x + k_2 y)$$

where s_i is the *i*th firm's sales and k_1 and k_2 are some positive parameters. If these two parameters are equal, market shares will be exactly equal to goodwill shares.

Observe the pioneer's market share. If $\alpha + \beta < 1$ then from equation (4) it can be easily computed that the pioneer's market share will be smaller than his goodwill share. The reverse is true if $\alpha + \beta > 1$. Thus in the case that $\alpha + \beta > 1$ a small increase in the pioneer's advertising and therefore goodwill will cause a larger then proportional increase in market share. Clearly this cannot be expected to be a stable situation.

For example, assume that the goodwill share of the pioneer is 60% and observe the following table:

	goodwill share	market share $\alpha + \beta = 0.6$	market share $\alpha + \beta = 1.4$
pioneer	60	56	64
follower	40	44	36

Thus changes in goodwill shares will tend to be magnified in market shares if $\alpha + \beta > 1$ and to contract if $\alpha + \beta < 1$.

The instantaneous gross profit of each firm, i.e., profits net of all costs except advertising, is given by:

$$\pi_i(p_1, p_2, x, y) = (p_i - c_i)s_i,$$
(5)

where c_i is the production cost of firm *i*, which might change over time. Experience effects are not taken explicitly into account in our model but rather exogenous changes of the cost function.

The standard condition that guarantees the existence of a unique pricing equilibrium is given by the following:

$$\pi_1^{p_1 p_1} \pi_2^{p_2 p_2} > \pi_1^{p_1 p_2} \pi_2^{p_2 p_1} \tag{6}$$

where superscripts denote differentiation with respect to the variable in question.

The payoff for firm *i* is defined as discounted net profits, i.e.,

$$J_i = \int_0^\infty e^{-rt} \{ \pi_i(p_1, p_2, x, y) - C_i(u_i) \} dt.$$
(7)

Each firm now wishes to maximize its own discounted profits by employing the optimal paths of pricing and advertising, given the choice of its rival. This, formally, is a non-cooperative differential game.

It is already well known that analyzing such games with structural dynamics involves many technical difficulties. The most general strategy space that one might consider in such games is the history-dependent strategies. However, even restricting the strategy space to be just functions of the current state variables (i.e., feedback or closed-loop, nomemory strategies) does not solve the problem of tractability. The feedback equilibrium is known to exist only for a small number of classes of differential games (see Fershtman 1987a for a discussion of such tractability problems). Thus since for the class of games under consideration in this paper the feedback equilibrium is not tractable we choose to analyze the equilibrium with open-loop strategies.³

Let the strategy set S_i be all piecewise continuous functions $u_i(t)$, $p_i(t)$ defined on $[0, \infty)$, that take their values in a compact set $[0, \bar{u}_i][c_i, \bar{p}_i]$ respectively. Such a strategy identifies for every time t an investment rate $u_i(t)$ and a price $p_i(t)$.

For every initial stock of goodwills x_0 and y_0 , define the game $G(x_0, y_0)$ as the game with strategy set S_i , payoff functions J_i , i = 1, 2; and at t = 0 the game starts at the initial stocks of $x(0) = x_0$ and $y(0) = y_0$, the function C_i is convex, twice differentiable and $C'_i(0) = 0$, and the function b_i is twice differentiable and bounded.

An open-loop Nash equilibrium for the game $G(x_0, y_0)$ is a pair of strategies $(u_1^*(t), p_1^*(t)), (u_2^*(t), p_2^*(t))$, such that $(u_i^*(t), p_i^*(t))$ maximizes J_i subject to (1.i) given $(u_j^*(t), p_j^*(t))$ for $j \neq i$.

A stationary Nash equilibrium is a pair of values (x^*, y^*) and a pair of strategies (u_1^*, p_1^*) ; (u_2^*, p_2^*) ; such that $u_1^* = \delta_1 x^*$, $u_2^* = \delta_2 y^*$ and (u_1^*, p_1^*) , (u_2^*, p_2^*) is a Nash equilibrium, for the game $G(x^*, y^*)$.

In a stationary Nash Equilibrium prices, advertising and goodwill levels are all constant with respect to time.

In solving for the closed-loop no-memory Nash equilibrium there are two known techniques:

- (i) Using the Pontryagin type conditions.
- (ii) The value function approach.

The first technique is not usable (in the nondegenerate case) since there is a "cross-effect term" in the necessary condition and this term makes the Pontryagin conditions intractable. The only case in which we can apply this technique is when this term is zero. However, when we discuss nonzero sum games the cross-term effect is zero only when the game is degenerate and definitely not in the model under discussion in this paper.

Using the value function approach: In this case we define the players' value functions which describe the players' expected discounted payoffs from the game as a function of the current state of the game. The value functions must satisfy the Hamilton-Jacobi-Bellman conditions, which form a system of partial differential equations. Unfortunately for most classes of differential games this system is not solvable and it is only proper to say that any advance in differential game theory is waiting for a breakthrough in the theory of partial differential equations. There are however some classes of games for which we can solve the Hamiltonian-Jacobi-Bellman equation. A common example is the class of linear quadratic games. Unfortunately the problem under investigation *cannot* be states as a linear quadratic game.

There are several other known classes of games for which it is possible to use the above methods to solve for the closed-loop equilibrium; for example a trilinear game, or a game for which the Hamiltonian is linear with respect to the control variable and exponential with respect to the state variables. However these classes of games are degenerate in the sense that they are tractable only because the open-loop equilibrium in these games is a special case of the closed-loop (feedback) equilibrium. See Fershtman (1987b) for a procedure that identifies such classes of games.

³ The *closed-loop no-memory* or *feedback* strategy space is a set of Markovian decision rules that specify at every t the player's action as a function of time and the observed state variables.

In the game we discussed above two sources of asymmetry are the initial conditions and the different cost functions. Before we move on we would like to be more specific regarding this asymmetry. The pioneer entered at time T_1 (see Figure 1). His initial production cost is c_1 . In his monopoly period he accumulates capital in the form of goodwill and also reduces his production cost. At some later date T_2 , a newcomer enters. His market share is a mirror image of the pioneer's market share, as depicted in Figure 1. His cost of production c_2 is larger than that of the monopolist at the entry date T_2 . At some later date T^* the cost of production of the two firms equate at a common cost c. (See Table 1.)

PROPOSITION 1. For every initial condition x_0 , y_0 , there exists an open-loop Nash equilibrium to the game $G(x_0, y_0)$, provided conditions (3) and (6) are met.

For a proof see Appendix 1.

In our simulation analysis presented in the next section we employ a specific demand function and show the implications of condition (6). It is clear that in case of identical gross profit functions, condition (6) is merely a concavity condition.

The uniqueness of equilibrium in the pricing game is essential to our analysis. This uniqueness enables us to use the reduced form and to express the profit function as a function of (x, y). Using such notation means that given (x, y) both firms charge the equilibrium prices which are uniquely defined. Nonuniqueness of the pricing game implies that we cannot use such a reduced form.

3. Stability: Long-Term Market-Share Reward

At the equilibrium each firm now chooses price and advertising paths that maximize its discounted net profits (5) subject to the constraints (1), given the paths of its rival. In the one-player case, and under our specific assumptions for each initial condition x_0 , there exists a unique optimal path. Moreover this path converges to a steady state, i.e., a point at which all three variables, price, advertising and goodwill, are stationary (see for example Gould 1970). In the two-players game there is no guaranteed uniqueness. Indeed, for any initial condition (x_0, y_0) there might be several Nash equilibria. The



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	Proposition	"Short-Term" Rewards as Long as Advantage Sustains	"Long-Term" Rewards Once Advantage Dissipates
1.	Production cost advantage leads to market-share rewards to pioneering brand	Validated (indirectly) by RF	Invalidated by FMM
2.	Advertising cost advantage leads to market-share rewards to pioneering brand	Validated by UCGM	Invalidated by FMM
3.	Lower price elasticity leads to market- share rewards to pioneering brand	Validated by RF	Invalidated by FMM
4.	Superior brand quality or position leads to market-share rewards to pioneering brand	Validated by RF, UCGM	Partially invalidated by RF
5.	Longer time lag between entry leads to long-term market-share rewards to pioneer brand		Invalidated by UCGM, FMM
6.	Longer time period in which pioneer held an advantage (1–4) leads to long term market share rewards to pioneering brand		Invalidated by FMM
7.	Order of entry per se affects long-term market share		Invalidated by RF, FMM. Partly validated by UCGM

 TABLE 1

 Propositions Relating to Market-Share Rewards

RF-Robinson and Fornell (1985).

UCGM-Urban, Carter, Gaskin and Mucha (1986).

FMM—Fershtman, Mahajan and Muller (this paper).

possibility that all of these will converge to a stationary point will be discussed shortly. In particular, notice that if one player decides to employ a cyclical advertising and consequently cyclical goodwill policy, it is straightforward to show that the best reply for the second firm is to adopt a cyclical policy as well.

We are interested in asymptotic stability of the equilibrium path, i.e., paths that approach a stationary equilibrium. A stationary equilibrium is defined, as in the one-player case, as an equilibrium of the game in which all variables are stationary. What we show is that even when the goodwill levels do not start at the stationary point, the system will converge towards this point as time tends to infinity.

Global stability implies that regardless of the initial condition (x_0, y_0) each equilibrium path of firms 1 and 2 converges to the same stationary level x^* and y^* .

Notice that global stability does not imply uniqueness. It is possible that from a particular initial condition there are several equilibria, each one of them converging to the stationary equilibrium. On the other hand, if from any initial conditions the equilibrium converges to a steady state which is the stationary equilibrium we can conclude that the stationary equilibrium is unique.

Given the goodwill accumulation game, standard analysis indicates that the stationary equilibrium is given by the following equations:

$$(r + \delta_1)C'_1(\delta_1 x) = \pi_1^x(x, y),$$
(8)

$$(r + \delta_2)C'_2(\delta_2 y) = \pi_2^y(x, y).$$
(9)

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This forms a nonlinear system of two equations with two unknowns x^* and y^* —which are the goodwill levels at the stationary equilibrium.

Note that the above system might have multiple solutions. None of the solutions depend on the initial conditions x_0 and y_0 .

The following condition guarantees not only the uniqueness of the stationary equilibrium but also that each equilibrium path converges to the stationary equilibrium, as is demonstrated in Proposition 2:

$$\alpha_i + \beta_i < 1. \tag{10}$$

PROPOSITION 2. The stationary equilibrium of the game $G(x_0, y_0)$ that satisfies (3), (6) and (10) is unique. Furthermore, the Nash equilibrium is globally asymptotically stable (GAS), i.e., from every initial condition, every equilibrium path converges to this stationary equilibrium. (For a proof see §4 and Appendix 2.)

In their 1985 paper, Robinson and Fornell identified several sources of market pioneer advantage. These sources fall into five main categories that relate the source to the following: price (or elasticity) differential, difference in cost of production, cost of advertising, quality and product line. Similar categories can be found in Robinson (1988) for industrial goods.

In this paper we deal with the first three sources (that relate to Robinson and Fornell's hypotheses 3 to 8), namely price, production and advertising.

Consider, for example, hypothesis 6 of Robinson and Fornell. It states that the pioneer, since it entered first, has absolute production cost advantage that leads to a higher market share.

We might accept this hypothesis in the short run, but in the long run the cost advantage will probably disappear due to technological diffusion, expiration of patent, unenforceability of patent, etc. In this case we would like to know whether the market-share advantage of the pioneer is permanent or temporary, i.e., will the difference in the market share disappear through time or will it remain positive throughout.

The answer given in the following proposition clearly points to the temporary nature of this advantage.

PROPOSITION 3. If the pioneer has an initial production or advertising cost advantage or a lower price elasticity of demand, but these differences vanish at some future, finite time, final market shares will not be dependent on the order of entry, the initial cost or elasticity advantage, or the time at which the elasticities or costs became equal. (See Appendix 3 for a proof.)

We thus find conditions under which initial advantages cannot be sustained.

There are, however, two main effects that will yield a long-term sustainable advantage and thus will make the pioneer's position preferable: (a) a relatively small market for a durable good, (b) nonreversible investments.

Both effects would alter the model's formulation and will yield a situation in which global asymptotic stability does not hold.

Assume that instead of equation (1) the dynamics are described by the following equations:

$$\dot{x}(t) = u_1(t)(N - x(t) - y(t)), \tag{1.1}$$

$$\dot{y}(t) = u_2(t)(N - x(t) - y(t)). \tag{1.2}$$

In this case there is a finite market potential N, and the investment is nonreversible in the sense that the capital level x or y can only increase. No decrease via depreciation is possible. In this case a firm that enters first, and accumulates capital beyond the steady state level (overcapitalization) cannot correct this situation once entry has occurred. Thus, whether or not it is in the best interest of the firm to divest, it cannot, and it will enjoy (or suffer) a higher market share throughout.

In equations (1.1)' and (1.2)' though the final total industry size is limited by N, i.e., $x(t) + y(t) \le N$, the division of industry sales between the two players may indeed depend on the order of entry and specifically on the amount of capitalization of the pioneer at the time of entry of the follower. It should be noted, though, that in this case x and y denote cumulative sales and not goodwill. Thus the model is not directly comparable to the one presented here.

4. Pattern and Speed of Convergence

We have shown in the previous section that once the pioneer's advantages dissipate by time T^* , long-term market share will not reflect these advantages. It is clear, however, that there is a carryover effect that is felt beyond T^* . The issue that we raise in this section is the effect of the model's parameters on the pattern and speed of convergence to the steady state.

Clearly having information on the time it takes the industry to converge to the neighborhood of the stationary equilibrium is essential when we evaluate the pioneering advantage. Late convergence implies that for a long time the pioneering firm has an advantage that translates itself in to higher profits. In analyzing such an advantage it is therefore important to find out how fast the industry converges to its steady state, and the pattern of this convergence.

In this section we examine several aspects of convergence. Because of limitations on analytical tractability, the issue of the speed of convergence is tackled via numerical methods. The pattern of convergence, however, is demonstrated via analytical treatment of the number of cycles it takes the equilibrium path to reach a given neighborhood of the stationary equilibrium and the relative size of each cycle (amplitude).

Let $g_i = (p_i - c_i)b_i(p_1, p_2)$, where p_i is the equilibrium price. Constructing the current value Hamiltonian and deriving the necessary conditions yield the following equation for the advertising path:

$$C_1''\dot{u}_1 = (r+\delta_1)C_1' - \alpha_1 x^{\alpha_1 - 1} y^{-\beta_1} g_1, \qquad (11)$$

$$C_2'' \dot{u}_2 = (r + \delta_2) C_2' - \alpha_2 y^{\alpha_2 - 1} x^{-\beta_2} g_2.$$
⁽¹²⁾

Graphically we can depict the equilibrium path by two phase diagrams as follows:



Observe that the $\dot{u}_1 = 0$ boundary in the (u_1, x) plane depends on the level of capital of the rival, i.e., y,

$$(r+\delta_1)C'_1 = \alpha_1 x^{\alpha_1-1} y^{-\beta_1}g_1.$$

Thus when y increases the boundary $\dot{u_1} = 0$ in the (u_1, x) plane moves down. Similarly for the $\dot{u_2} = 0$ in the (u_2, y) plane. Thus the paths have to reach the stationary equilibrium at *exactly* the same time. Since if, say, y reaches the stationary equilibrium before x does, the movement of x causes the $\dot{u_2} = 0$ boundary and thus the equilibrium point itself to move in the (u_2, y) plane. Thus our analysis of the speed and pattern of convergence will be carried out by examining the behavior of the boundaries $\dot{u_i} = 0$ and their relationship with changes of x and y.

Any converging path can be cyclic or monotonic. If it is *cyclic* then from a certain time T^* the path is not monotonic but converges in cycles that are decreasing on their amplitude, i.e., a *damped* series of cycles. We will call a path *monotonic* if it is monotonic from a certain time T^* onwards.

The Cyclic Case

In order to prove convergence we will show that the series of cycles is damped such that the amplitude of one cycle cannot exceed a certain percentage of the amplitude of the previous cycle. Moreover the dampening factor is proven to be exogenous (i.e., it does not depend on the cycle itself), which guarantees convergence. The proof here is different from the argument in our previous analysis (Fershtman and Muller 1986), since in this case the condition that $|\Pi^{xx}| > |\Pi^{yy}|$ does not hold. Moreover, in order to see the effect of the parameters on the dampening factor we need a relation between the amplitude of one cycle of x and its predecessor. This makes the proof more complex.

Consider the symmetric case, and two consecutive cycles a and b of the capital of the pioneer (x). Times t_a and t^a are the start and end times for cycle a and similarly for t_b and t^b :



Specifically we show that the following inequality holds (see Appendix 2):

$$|x(t^{b}) - x(t_{b})| < (1 - \epsilon)^{2} |x(t^{a}) - x(t_{a})|.$$
(13)

The dampening factor, ϵ , is given by equation (14) (see Appendix 4) and its relation to the model parameters will be discussed shortly.

$$1 - \epsilon = \beta / (1 - \alpha + \delta (r + \delta) / g \alpha).$$
(14)

We have therefore shown that the size of the dampening factor ϵ determines the pattern of convergence of the path to the steady state.

With a *large* ϵ , it is clear that it would take *less* cycles to arrive at a given neighborhood of the steady state. From equation (14), the following proposition is apparent:

PROPOSITION 4a. Let the game $G(x_0, y_0)$ satisfy conditions (3), (6) and (10). The smaller the capital intensity parameter α , the competitive parameter β or the gross profit

factor g, and the larger the discount rate r or the decay parameter δ , the less cycles the equilibrium path takes to arrive at a given neighborhood of the stationary equilibrium.

The Monotonic Case

Convergence in this case is easily achieved by noting that our assumptions on the cost and revenue function rule out convergence of the path to zero or divergence to infinity. Therefore the path converges to some stationary equilibrium. Since by condition (10) the stationary equilibrium is unique, the path converges to (x^*, y^*) , the unique stationary Nash Equilibrium.

With respect to speed and pattern of convergence, if the path is monotonic i.e., it is monotonic from a certain point of time T^* , the treatment of the cycles up to T^* precisely follows our previous discussion. If it is monotonic from the initial time $t_0 = 0$ we cannot use the previous analysis, since it relies on the interlacing argument and in particular on the fact that a cycle of one path (x) determines the boundaries of a cycle of the other path (y). What we can tell about this purely monotonic case, apart from the simulation analysis, is the distance the path has to cover from its initial point to the steady state, or in other words the variability of the path between its two extreme points. This is done by measuring the distance $|x^* - x_0|$, and performing a parametric analysis on it. This is valid, of course, only in the purely monotonic case, since in any nonmonotonic convergence, the distance the path covers is potentially larger than $|x^* - x_0|$. Computing this variable yields the following equation:

$$|x^* - x_0| = |(g\alpha/\delta(r+\delta))^{1/(2-\alpha+\beta)} - x_0|.$$
(15)

A straightforward differentiation of equation (15) proves the following proposition:

PROPOSITION 4b. Let equilibrium of the game $G(x_0, y_0)$ that satisfies conditions (3), (6) and (10) be monotonic, and let $x_0 < x^*$.

The smaller the capital intensity parameter α , or the gross profit factor g, and the larger the discount rate r, the decay parameter δ or the competitive parameter β , the smaller is the distance the goodwill path covers over the entire planning horizon.

It should be noted that this measure is somewhat different from the size or number of cycles in the nonmonotonic case, which might explain the variation in the effect of β .

Using a simulation study we directly checked the speed of convergence using the following method: Following Wolf and Shubik (1978), McGuire and Staelin (1983), and Eliashberg and Jeuland (1986), we assume a linear purchase rate equation. That is,

$$b_i(p_1, p_2) = a_i(1 - kp_i) + \gamma(p_j - p_i), \quad j \neq i.$$
 (16)

It is cumbersome but straightforward to check that if $1 > kc_i$, where c_i is the production cost of firm *i*, then demand is positive and the monopolist's price will be larger than its cost c_i , and if $\gamma < a_i k$ then the oligopolist price will be larger than its cost. Since we assume symmetry (other than initial advantage) we assume, without loss of generality, that $a_1 = a_2 = \frac{1}{2}$. It is also easily checked that condition (6) is satisfied and that there exists a unique solution for the price paths.

Since the simulation uses a finite horizon problem, we have set $u_i(T) = u^*$, where T is the planning horizon, and u^* is the equilibrium advertising level. The setting of the final advertising level is done so that end game considerations will not come into play. This indeed approximates the infinite horizon game much better than with final conditions of $u_i(T) = 0.4$

⁴ We are indebted to an anonymous referee for pointing out this issue to us. This approximation requires that T be large enough so that convergence to the stationary equilibrium is made possible.

The analysis was done on the discrete analog of equations (1.1), (1.2), (11) and (12) that form a boundary value problem with four unknowns and four equations. It was run on the Lotus 1-2-3 program, using a first-order (Euler Cauchy) solution. In such games it is easy to check that the solution generated by the program is indeed an equilibrium since at equilibrium each firm employs the best policy in terms of net present value (equation (7)) against the policy of its rival. Thus changes in the solution of *one* of the players yields a lower NPV for this particular player.

A planning horizon of 50 periods proved long enough for the market shares to converge towards the 50 percent mark with an error of 1 percentage point (except for one case in which the error was 1.5 percent). The cost functions were set to be quadratic (and identical).

The range of parameters is as follows: we have taken a base case on which $\alpha = 0.75$, $\beta = 0.2$, $\delta = 0.06$, r = 0.02, c = 20, $k = \frac{1}{100}$, $\gamma = \frac{1}{300}$. Thus we have assumed symmetry with respect to all parameters. We then varied the parameters systematically and observed the convergence of the pioneer's market share at periods 5, 10, 20 and 50. Since all asymmetries such as production cost advantage impact goodwill level, in this simulation we have considered the direct effect of asymmetric initial levels of goodwill. This was done by having the pioneer's initial goodwill level set at $x^*/2$, and the follower's at $x^*/100$, where x^* is the (common) equilibrium level of goodwill.

We define the speed of convergence in a particular case to be faster than in another case, if the levels of market shares in the first case are closer to the final equilibrium level than in the second case *at all four periods* (5, 10, 20 and 50). The results are shown in Table 2, and in the following proposition:

PROPOSITION 4c. Let the game $G(x_0, y_0)$ satisfy conditions (6) and (10) and let the purchase rate equation be linear as in (16), so that condition (6) is satisfied. The results of the simulation study demonstrate that the smaller the capital intensity parameter α ,

Speed of Convergence							
	Market Share of Pioneer in Period						
Parameter	5	10	20	50			
$\alpha = 0.35$	56.07	53.13	51.17	50.13			
$\alpha = 0.4$	57.2	53.73	51.41	50.14			
$\alpha = 0.5$	59.52	55.05	51.96	50.22			
$\alpha = 0.6$	62.25	56.57	52.63	50.29			
$*\alpha = 0.75$	66.62	59.26	53.91	50.51			
$\beta = 0.05$	63.76	57.26	52.82	50.32			
$\beta = 0.1$	64.67	57.88	53.15	50.36			
$\beta = 0.15$	65.62	58.55	53.51	50.44			
$*\beta = 0.2$	66.62	59.26	53.91	50.51			
$\delta = 0.04$	71.35	63.29	56.72	51.54			
$\delta = 0.06$	66.62	59.26	53.91	50.51			
$\delta = 0.1$	61.07	55.11	51.49	50.02			
$\delta = 0.2$	55.02	51.42	50.14	50.00			
*r = 0.02	66.62	59.26	53.91	50.51			
r = 0.06	66.07	58.98	53.81	50.49			
r = 0.1	65.69	58.78	53.74	50.49			
<i>r</i> = 0.2	65.06	58.48	53.64	50.48			

TABLE 2

* The base case is denoted with a star.

and the competitive parameter β , and the larger the discount rate r or the decay parameter δ , the faster is the convergence to the stationary equilibrium. It should be noted that changes in the parameters affecting the profit margin (i.e., k, c, and γ) did not produce any changes in the rate of convergence.

We can thus summarize the effects of the model parameters on the pattern and speed of convergence as follows: with an increase in r or δ , or a decrease in α and β we find either analytically or via simulation that the number of cycles and the size of each cycle decline, and the convergence to the steady state is faster. The directional effect of the profit margin (g) on the pattern of convergence is the same as α or β .

5. Anticipation of Entry

Eliashberg and Jeuland (1986) provided a detailed and thorough analysis of a pricing game similar in nature to our advertising game. One of their main points of research is to investigate the behavior in terms of price path of a monopolist who does not anticipate entry (surprised monopolist) versus a far-sighted monopolist who correctly anticipates entry.

We ask the same question with respect to advertising and goodwill paths. Specifically, consider the case in which the industry starts as a monopoly and at some future date T_2 a newcomer enters. The far-sighted monopolist plans in full knowledge of this date, while the surprised monopolist does not know or foresee such an occurrence.

The surprised monopolist maximizes the following:

$$\int_0^\infty \left[(p_1 - c_1) b_1(p_1) x_m^\alpha - C_1(u_{1m}) \right] e^{-rt} dt \tag{17}$$

subject to equation (1).

We wish to compare this plan to the plan of a monopolist who anticipates entry at time T_2 and thus maximizes the following:

$$\int_{0}^{T_2} \left[(p_1 - c_1) b_1(p_1) x^{\alpha} - C_1(u_1) \right] e^{-rt} dt$$
(18)

subject to equation (1), and, from time T_2 , he engages in a game in which the equilibrium is one in which he maximizes the following:

$$\int_{T_2}^{\infty} \left[(p_1 - c_1) b_1(p_1, p_2) x^{\alpha} y^{-\beta} - C_1(u_1) \right] e^{-rt} dt \tag{19}$$

given the path of his rival (the newcomer) and equations (1.1) and (1.2). For convenience we let the function $b_1(p_1, p_2)$ be defined as in the previous section (equation (16)) and $b_1(p_1)$ be the same function with γ set to zero. Clearly the arguments in this section will hold for any demand function satisfying condition (6).

PROPOSITION 5. For $0 \le t \le T_2$ the advertising level of the surprised monopolist is larger than that of the farsighted monopolist, i.e., $u_{1m}(t) > u_1(t)$, $0 \le t \le T_2$. (See Appendix 5a for a proof.)

Thus we have shown that the surprised monopolist advertises up to T_2 more than the far-sighted one and thus builds more goodwill; that is, he *overcapitalizes*, as compared to a monopolist who correctly anticipates entry. The reason is that the surprised monopolist plans to accumulate goodwill to reach a final level that is higher than he is going to reach eventually. This is so since the steady-state level of goodwill of a monopolist is higher than that of *each* of the oligopolists (see Appendix 5b), and the surprised monopolist plans his advertising path as if he were going to remain a monopolist for the

duration of the game. The monopolist who correctly anticipates entry, knows that the final level of goodwill he is going to reach is lower and plans accordingly, i.e., advertises less during his monopoly period.

An outcome of this overcapitalization of the surprised monopolist, is that his market share will be higher than that of the farsighted monopolist both in the monopoly and in the duopoly period.

6. Conclusions and Managerial Implications

In this paper we have examined the long-term effects of the order of entry into an industry. What we find is that under some simple concavity conditions, final market shares in an industry in which a pioneer entered first and a newcomer later, do not depend on the order of entry or on the length of the monopoly period. If, in addition to higher level of goodwill, the pioneer initially also enjoys the advantages of lower price, brand loyalty, lower production cost and lower advertising costs, which dissipate at some later date, the final market shares do not depend on the magnitude of these advantages, or on the length of time it took the newcomer to overcome them. The speed and pattern of the disappearance of the pioneer's market-share advantage depend on the specific conditions as represented by the model's parameters.

When we investigate the question of anticipation of entry we find that a monopolist who does not anticipate entry (surprised) overcapitalizes as opposed to one who correctly anticipates entry. The reason is that the surprised monopolist is accumulating for a final level that is higher than the one he is eventually going to realize.

How do these theoretical findings relate to recent empirical studies in marketing and especially to Robinson and Fornell (1985) and Urban, Carter, Gaskin and Mucha (1986)? What we suggest is that the mere order of entry has no relevance to the market share in the long run. It is the effect the order of entry has on production costs, advertising costs, price elasticity and, by implication, quality, distribution and breadth of line that matters. These latter variables, if their advantage is permanent, affect market share.

This is fully supported by the empirical findings of Robins and Fornell (1985) and partially by Urban et al. (1986). In Robinson and Fornell, pioneers, on average, had a higher market share than early followers, who, in turn, had higher market shares than late followers. However, the difference is explained via the effect of the order of entry on four variables: quality, line breadth, price and costs. Indeed Table 4 of Robinson and Fornell indicates that the order of entry (e.g., "pioneer" versus "earlier follower") has a significant effect on these four variables, while it has *no* significant direct effect on market share.

In Urban et al., the order of entry *per se* has a significant effect on market share. They considered, however, the effect of only three additional variables: product positioning, advertising and time lag between entry. Indeed, the first two have significant effects while the latter does not.

This last finding about time lag between entry supports ours, while the first one does not. It is possible that had more explanatory variables, such as production costs differential, been added to the analysis, the direct effect of the order of entry would have become nonsignificant. The main findings of these two works that relate to ours are summarized in Table 1. Thus what we have done in this paper is to re-examine the long-term validity of the propositions of Table 1.

Two theoretical findings not tested in these works are that once the cost or demand elasticities differential vanishes, market share advantage disappears as well, and that the time it takes this differential to vanish has no significant effect on final market share.

What are the managerial implications of our results?

First and foremost they reduce the importance of being first with a new innovation and therefore first in a new market. Thus when allocating funds for R&D purposes the stakes at the end of that activity depend on the timing of introduction to a lesser degree than is usually thought. Thus when considering an "R&D game" in which the aim is to be the first with a new innovation, one should consider the long-run market position and profitability implications of being first, which are much less affected by the outcome of the R&D game than the short-run market position and profitability.

The transitory nature of these advantages was recently highlighted by Lieberman and Montgomery (1988) who state that "It is now generally recognized that diffusion occurs rapidly in most industries and learning-based advantages are less widespread than was commonly believed in the 1970s."

Second, consider an innovator who believes that being first in the market yields longterm sustainable advantages in terms of market share and profitability. At the R&D stage he overcapitalizes in terms of high R&D expenditures needed to assure him a pioneer position. Once he enters, he again overcapitalizes since he does not correctly foresee entry. His overall return on investment (ROI) will be low even if his profits are high simply because of a high capital level (the denominator in the ROI ratio).

What are the shortcomings of our approach? First we deal with a duopoly setting, i.e., a two-firm case. The extension to the multifirm case is rather straightforward. There is nothing in the model or the mathematics to prevent such extension. Indeed our initial model (Fershtman and Muller 1984) was extended by Dockner and Takahashi (1988a, b) to the n-players case.

A more basic shortcoming is the assumption of the homogeneity of the goodwill variable. Since publication of the work by Bass (1969), researchers have extended the notion of goodwill by noting that goodwill is not a homogeneous capital asset. Models such as TRACKER (1978), NEWS (1982), and the one proposed by Dodson and Muller (1978) have broken down goodwill to three and more components, from awareness to final adoption. None of this can be reflected in a paper where we assume homogeneity of goodwill and advertising. This homogeneity assumption might be crucial to our argument in the case of a durable good. If, for example, consumers who are innovators adopt the durable product first, and they are few in number, the pioneer will enjoy the benefits that these innovators bring along, mainly their relatively high word-of-mouth coefficient. Latecomers will have to be content with less effective groups. These groups such as early and late majority are inferior in terms of their opinion leadership, social involvement and other variables that all sum up to the word-of-mouth coefficient. This will certainly have a short-term effect, and it might have a long-term effect as well.

We have not addressed the possibility of a new technology as well. If, at some point of time, a new technology emerges, this might profoundly affect the structure and nature of the game. If, for example, this technology has emerged during the initial monopolist period, it is likely, if any fixed costs are involved, that the newcomer will have an advantage over the pioneer.

An avenue for future research could incorporate the above considerations into a framework that will address the issue of long-term market-share rewards in an integrated fashion.⁵

⁵ We would like to thank Mark Satterthwaite, Jehoshua Eliashberg, an Associate Editor of *Management Science* and three anonymous referees for a number of helpful comments and suggestions. Computational assistance by Yavin Muller is gratefully acknowledged.

Appendix 1. Proof of Proposition 1

The proof is based on Theorem 1 in Fershtman and Muller (1984). We break the game (from time T_2 when the newcomer has entered) into two periods. From T_2 to T^* in which the cost of the newcomer is slowly decreasing and the period from T^* on in which the costs are unchanged and equal. The existence of equilibrium

for the second part of the game follows directly from Fershtman and Muller (1984). For the first part of the game the proof in the above paper has to be modified as follows:

Let $g_i(t) = [p_i(t) - c_i(t)]b_i(p_1(t), p_2(t))$ where $p_i(t)$ is the equilibrium time path of price of firm *i*, whose existence is assured by condition (6).

Let $R_1(x, y) = x^{\alpha}y^{-\beta}$ and $R_2(x, y) = y^{\alpha}x^{-\beta}$. For convenience we have rescaled the goodwill levels so that their common lower bound is one instead of zero (see footnote 2). This will ensure the boundedness of R_i . The solution of x(t) is given by:

$$x(t) = x_0 e^{-\delta_1 t} + \int_0^t e^{-\delta_1 (t-s)} (C_1')^{-1} \left\{ \int_s^{T^*} g_1(\tau) \alpha x(\tau)^{\alpha-1} y(\tau)^{\beta} e^{-(r+\delta_1)(\tau-s)} d\tau \right\} ds$$

and similarly for y(t).

The rest of the proof of Theorem 1 in Fershtman and Muller (1984) follows if $g_i(t)$ is bounded. The function $g_i(t)$ is indeed bounded since $p_i(t)$ is (see the definition of the strategy space) and since b_i is bounded by assumption.

Appendix 2. Proof of Proposition 2

(a) Uniqueness of the Stationary Equilibrium. Define $x = h_1(y)$ and $y = h_2(x)$ as the solutions of (8) and (9), respectively. Differentiating equation (8) we have:

$$h_1' = \pi_1^{xy} / (\delta_1(r + \delta_1)C_1'' - \pi_1^{xx})$$

and similarly for h'_2 from equation (9). Since π_1^{xx} and π_2^{yy} are negative, the sign of h' is the same as the sign of π_i^{xy} .

Since $\pi_1^{xy} < 0$, it suffices to show that at every stationary point $(h_1^{-1})' < h_2'$. Computing these derivatives yields the following condition:

$$(\delta_1(r+\delta_1)C_1''-\pi_1^{xx})(\delta_2(r+\delta_2)C_2''-\pi_2^{yy})>\pi_2^{xy}\pi_1^{xy}.$$

It is straightforward to compute that for the above inequality to hold the following condition on α_i and β_i is sufficient: $(1 - \alpha_1)(1 - \alpha_2) < \beta_1\beta_2$. A sufficient condition for the above is the following simple condition: (inequality (10)) $\alpha_i + \beta_i < 1$.

(b) Global Asymptotic Stability. The proof continues §4, equations (11)–(13). Let $x = f_1(y)$ denote the level of capital at the intersection of the $\dot{u}_1 = 0$ and the $\dot{x} = 0$ in the (u_1, x) plane.

Define $y = f_2(x)$ symmetrically. When the path $(x(t), u_1(t))$ is in the region in which $\dot{x} < 0$ it cannot cross the $\dot{x} = 0$ boundary unless the $\dot{u}_1 = 0$ boundary is below the path. In the same way, when the path is in the $\dot{x} > 0$ region it cannot cross the $\dot{x} = 0$ line unless the $\dot{u}_1 = 0$ boundary is above the path. Thus we have:

$$|x(t_b) - x(t^b)| < |f_1(y(t_b)) - f_1(y(t^b))|.$$

Since f_1 is a continuous function on a compact set (t_a, t^b) it achieves a maximum and a minimum at \overline{t} and \underline{t} respectively.

Observe the following string of inequalities:

$$\begin{aligned} |x(t_b) - x(t^b)| &< |f_1(y(t_b)) - f_1(y(t^b))| \\ &\leq \max_{t_a \leq t_1, t_2 \leq t^b} \{ |f_1(y(t_1)) - f_1(y(t_2))| \} \\ &= |f_1(y(\bar{t})) - f_1(y(\underline{t}))| \\ &= |\partial f_1(\xi_1) / \partial y| |y(\bar{t}) - y(\underline{t})| \\ &< |\partial f_1(\xi_1) / \partial y| |f_2(x(\bar{t})) - f_2(x(\underline{t}))| \\ &= |\partial f_1(\xi_1) / \partial y| |\partial f_2(\xi_2) / \partial x| |x(\bar{t}) - x(\underline{t})| \\ &\leq \max_{\xi_1, \xi_2} \{ |\partial f_1(\xi) / \partial y| |\partial f_2(\xi) / \partial x| \} |x(\bar{t}) - x(\underline{t})| \\ &< (1 - \epsilon)^2 |x(\bar{t}) - x(\underline{t})| \\ &\leq (1 - \epsilon)^2 |x(t^a) - x(t_a)|. \end{aligned}$$

The first three inequalities were previously discussed. The next equality uses the mean value theorem where ξ_1 is some intermediate value in the region $[\bar{t}, t]$. The fifth line requires the property of f_2 used in the first inequality. Note that in order to establish this inequality we need that y(t) achieves a maximum and a minimum at t and \bar{t} respectively. This follows our phase diagram analysis since when y increases, the boundary $\dot{u}_1 = 0$ in the

 (u_1, x) plane moves down (and vice versa). Thus if $f_1(y(t))$ is maximal at \overline{t} , then y(t) is minimal at that point. We then use the mean value theorem again. In Appendix 4 we show that:

$$\partial f_1(\xi)/\partial y | |\partial f_2(\xi)/\partial x | < (1-\epsilon_1)(1-\epsilon_2) < (1-\epsilon)^2$$

where ϵ_1 is given by:

$$1 - \epsilon_i = \beta_i / (1 - \alpha_i + \delta_i (r + \delta_i) / g_i \alpha_i)$$

and $\epsilon = \text{Min } \epsilon_i$.

The last inequality follows from the fact that *a* and *b* are two consecutive cycles and therefore share the same maximum (or minimum), and from the fact that prior to establishing the last inequality we already have that $|x(t_b) - x(t^b)| < (1 - \epsilon^2)|x(\bar{t}) - x(\underline{t})|$. Thus both the minimum and the maximum *cannot* occur in the interval (t_b, t^b) . Q.E.D.

Appendix 3. Proof of Proposition 3

For every $T_1 < T_2$, the first period up to T_2 is a monopoly period, and the duopolistic game starts at T_2 . The difference between the duopolistic game in which the pioneer enters at T_1 , and the game in which he enters at T_2 , is only in the initial conditions of the duopolistic game. The global asymptotic stability property implies that the final goodwills do not depend on the initial conditions. It follows that $MS_i^*(T_1, T_2) = MS_i^*(T_2, T_1)$. Thus the order of entry has no effect on the final market shares.

Denote by T^* the time at which the last remaining advantage of the pioneer has vanished. At this time the market shares of the two firms are $MS_1(T^*)$ and $MS_2(T^*)$, which correspond to goodwills of $x(T^*)$ and $y(T^*)$. We now start the game described in §2 at time T^* , i.e., translate time t to time $t - T^*$. The new game will converge to final market shares MS_1 and MS_2 that do not depend on $x(T^*)$ and $y(T^*)$ or on $MS_1(T^*)$ and $MS_2(T^*)$.

Now note that the calendar time does not have any effect on the game. That is, via the translation $t - T^*$ we start the game at time zero, and the game is identical to a game for which we made a translation of $t - T^{**}$, where T^{**} is a (different) time at which the production costs of the two firms become equal. Q.E.D.

Appendix 4. Computation of ϵ

We show in this appendix that if $x = f_1(y)$ denotes the level of capital at the intersection of $\dot{u}_1 = 0$ and $\dot{x} = 0$, then $|\partial f_1/\partial x \partial f_2 \partial y| < (1 - \epsilon)^2$. Using equations (11), (12) and (5) we can calculate the following expression:

$$\begin{aligned} |\partial f_1 / \partial x \partial f_2 / \partial y| &= \frac{\Pi_1^{xy}}{\delta_1 (r + \delta_1) - \Pi_1^{xx}} \frac{\Pi_2^{xy}}{\delta_2 (r + \delta_2) - \Pi_2^{xx}} \\ &= \frac{(\Pi_1^{xy} \Pi_2^{xy}) / \Pi_1^{xx} \Pi_2^{yy}}{(1 - \delta_1 (r + \delta_1) / \Pi_1^{xx})(1 - \delta_2 (r + \delta_2) / \Pi_2^{yy})} \end{aligned}$$

This last equality was achieved by dividing throughout by $-\Pi_1^{xx}$ and $-\Pi_2^{yy}$. Calculating the derivatives of Π_i we achieve the following:

$$|\partial f_1/\partial x \partial f_2/\partial y| < \frac{\beta_1/(1-\alpha_1)}{(1+\delta_1(r+\delta_1)/g_1\alpha_1(1-\alpha_1))} \frac{\beta_2/(1-\alpha_2)}{(1+\delta_2(r+\delta_2)/g_2\alpha_2(1-\alpha_2))}.$$

Since $\alpha_i < 1$ and $\alpha_i + \beta_i < 1$, this last expression is less than 1. Thus $|\partial f_1/\partial x \partial f_2/\partial y| < (1 - \epsilon)^2$ where $\epsilon = \text{Min } \epsilon_i \text{ and } 1 - \epsilon_i = \beta_i/(1 - \alpha_i + \delta_i(r + \delta_i)/g_i\alpha_i)$. In the symmetric case ϵ is clearly the expression given in equation (14).

Appendix 5. Goodwill Accumulation of Surprised Monopolist and of a Monopolist Who Correctly Anticipates Entry

(5a) Let firm 1 be the pioneer and firm 2, the newcomer. Let $g_m = (p_1 - c_1)b_1(p_1)$ with p_1 being the monopolist price, and $b_1(p_1) = (1 - kp_1)/2$. Let $g_c = (p_i - c_i)b_i(p_1, p_2)$ where b_i is defined in equation (16) and p_i is the equilibrium price. It is straightforward to check that g_m is larger than g_c , as expected.

Constructing the current value Hamiltonian and deriving the necessary conditions yield that the monopolist who anticipates entry plans according to:

$$C_1''\dot{u}_1 = (r + \delta_1)C_1' - \alpha x^{\alpha - 1}g_m \quad \text{for} \quad 0 \le t \le T_2, \tag{A.1}$$

$$C_1''\dot{u}_1 = (r+\delta_1)C_1' - \alpha x^{\alpha-1}y^{-\beta}g_c \quad \text{for} \quad T_2 \le t \le T$$
 (A.2)

and $u_1(T_2) = u_1(T_2)$ (the continuity of u_1 at T_2 follows from the continuity of the multiplier at T_2).

The surprised monopolist plans as if no entry will occur and thus his path is planned according to:

$$C_1'' \dot{u}_{1m} = (r + \delta_1) C_1' - \alpha x_m^{\alpha - 1} g_m.$$
(A.3)

Observe the following figure of advertising paths:



Case 1. $u_{1m}(0) > u_1(0)$. The paths of u_1 and u_{1m} cannot intersect before time T_2 . This is true since for an intersection of the two paths to occur, it has to hold that: $\dot{u}_{1m} < \dot{u}_1$ at the intersection time. Since up to that time $u_{1m}(t) > u_1(t)$, it follows from equation (1) that $x_m(t) > x(t)$. Since $\alpha < 1$, and at the intersection time, $u_{1m} = u_1$, it follows that $\dot{u}_{1m} > \dot{u}_1$ —a contradiction.

Case 2. $u_{1m}(0) < u_1(0)$. The above argument, being symmetric in u_{1m} and u_1 shows that the paths cannot intersect before T_2 , thus $u_{1m}(t) < u_1(t)$ for $0 \le t \le T_2$. The paths however cannot intersect after T_2 since at the intersection time, $\alpha x^{\alpha-1} y^{-\beta}g_c < \alpha x_m^{\alpha-1}g_m$ since $x^{\alpha-1} < x_m^{\alpha-1}$, $y^{-\beta} < 1$ since y > 1 and $\beta > 0$, and $g_c < g_m$. The same argument holds for all $T_2 \le t < \infty$. Observe that at the steady state, $x_m^* = x^*$, since once entry occurs we have exactly the same game in both situations (farsighted vs surprised), but with different initial conditions; global asymptotic stability implies these levels do not depend on initial conditions and so $x_m^* = x^*$. However, since $u_{1m}(t) < u_1(t)$ for all $t, 0 \le t \le \infty$, and since $x_m(T_2) < x(T_2)$ the following inequality holds:

$$x_m^* = x_m(T_2) + \int_{T_2}^{\infty} e^{-b(t-s)} u_{1m}(s) ds < x(T_2) + \int_{T_2}^{\infty} e^{b(t-s)} u_1(s) ds = x_1^*.$$

This contradicts the fact that $x_{1m}^* = x_1^*$.

(5b) From equations (A.1) and (A.2), the steady state level of the monopolist is the solution of the following:

$$x_m^{1-\alpha}C_1'(\delta x_m) = g_m \alpha/(r+\delta), \tag{A.4}$$

while the level of the oligopolist is given by the solution of the following:

$$x^{1-\alpha}C_1'(\delta x) = g_c y^{-\beta} \alpha / (r+\delta).$$
(A.5)

Since $\alpha < 1$, and $C''_1 > 0$, the L.H.S. of equations (A.4) and (A.5) are increasing in x. Since $g_m > g_c y > 1$ and $\beta > 0$, it follows that $x^m > x$.

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