

## DIFFUSION OF NEW PRODUCTS: EMPIRICAL GENERALIZATIONS AND MANAGERIAL USES

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The diffusion model developed by Bass (1969) constitutes an empirical generalization. It represents a pattern or regularity that has been shown to repeat over many new products and services in many countries and over a variety of circumstances. Numerous and various applications of the model have led to further generalizations. Modifications and extensions of the model have led to further generalizations. In addition to the empirical generalizations that stem from the model, we discuss here some of the managerial applications of the model.

**(Diffusion; Forecasting; New Product Research)**

### 1. Introduction: The Generalization

An empirical generalization has been defined by Bass (1993, 1995) as “a pattern or regularity that repeats over different circumstances and that can be described simply by mathematical, graphic, or symbolic methods.” The description may be approximate rather than exact, and the pattern need not always hold.

An empirical generalization that satisfies this definition is the S-shaped pattern that has been observed in data depicting the natural growth of many phenomena as diverse as the future populations of cars and computers, the life expectancy of creative geniuses (e.g., Mozart), the frequency of economic booms and busts, the birth rate among women, the number of fatal car accidents, the incidence of major nuclear accidents, and deaths from AIDS (Modis 1992).

The theoretical model leading to empirical support for the existence of the S-shaped pattern to represent the first-purchase growth (cumulative distribution of adoption over time) for new products in marketing was first presented by Bass (1969). Unlike other growth studies in the physical or social sciences, however, that often do not concern themselves with the underlying processes that generate the S-shaped regularity, Bass relied on diffusion theory to model the timing of adoption that leads to a particular S-shaped growth pattern for new products or technologies. Hence, the new product *growth* model suggested by Bass is popularly known as the Bass new product *diffusion* model. Note that diffusion models are growth models but the reverse may not be true.

As a theory of communications, diffusion theory's main focus is on communications channels, which are the means by which information about an innovation is transmitted to or within the social system (Rogers 1983). These means consist of both the mass

media and interpersonal communications. Members of a social system have different propensities for relying on mass media or interpersonal channels when seeking information about an innovation. Interpersonal communications, including nonverbal observations, are important influences in determining the speed and shape of the S-shaped pattern of the diffusion process in a social system.

The central proposition of the Bass diffusion model is that:

$$\begin{aligned} & \text{The probability of adoption at time } t \text{ given that adoption has not yet occurred} \\ & = p + q \text{ cumulative fraction of adopters at time } T. \end{aligned}$$

If  $f(t)$  is the probability of adoption at time  $t$  and  $F(t)$  is the cumulative distribution, the proposition may be written as:

$$f(t)/[1 - F(t)] = p + qF(t). \quad (1)$$

Alternatively, (1) may be written as:

$$n(t)/[m - N(t)] = p + (q/m)N(t), \quad (2)$$

where  $m$  represents the market potential,  $n(t)$  is the number adopting at time  $t$  ( $n(t) = mf(t)$ ), and  $N(t) = mF(t)$  is the cumulative number of adopters at time  $t$ . Equation (2) implies the differential equation:

$$n(t) = [m - N(t)][p + (q/m)N(t)] = pm + (q - p)N(t) - (q/m)[N(t)]^2. \quad (3)$$

A closed-form solution exists for this equation in the time domain that is equivalent to the expression in equation (3) for the cumulative adoption domain. For purposes of understanding the time-related behavior of the adoption pattern as related to parameter variation as well as for many forecasting applications, the existence of the closed-form solution is important.

Bass termed  $p$  and  $q$  as the coefficients of innovation and imitation, respectively, to capture the proportional adoptions in  $f(t)$  due to “innovators,”  $p(1 - F(T))$ , and interpersonal communications,  $qF(t)(1 - F(t))$ . The coefficients  $p$  and  $q$  have also been termed as the coefficient of external influence and internal influence (Mahajan et al. 1990). Figure 1 shows the graphics of the new product adoption and growth patterns captured by the Bass diffusion model.

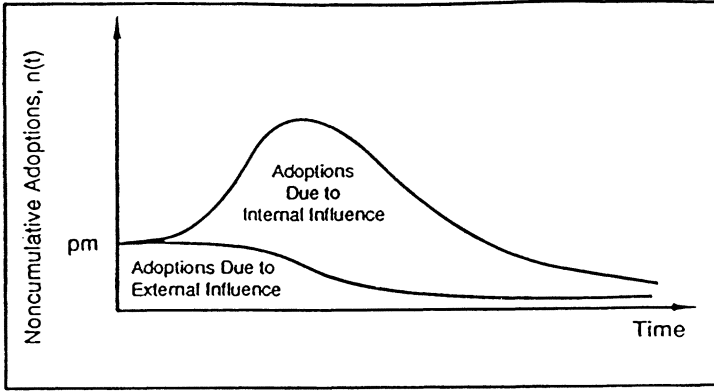
## 2. The Conditions

The model is designed to apply to data that satisfy certain conditions. We briefly discuss these below.

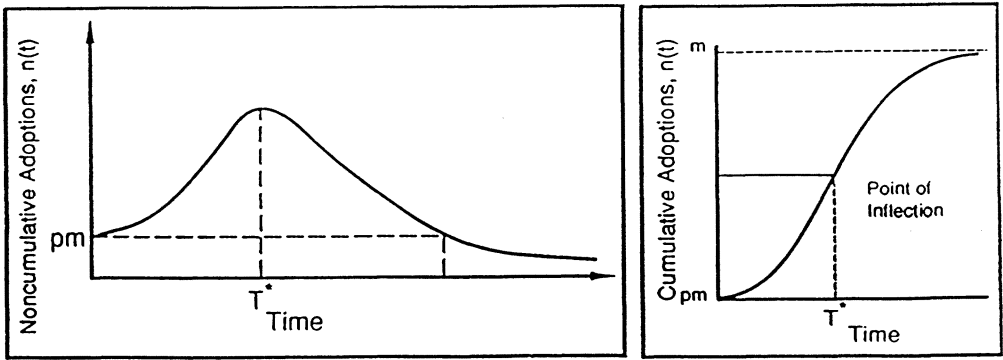
### 2.1. First Purchase Demand

The model is designed to apply to initial purchase (adoption) of the product and not to apply to replacement demand. Over time the replacement component of total demand will increase relative to the initial purchase component; and if sales data, as opposed to adoption data, are used in fitting the model, care must be taken to restrict time periods to those in which the replacement demand is negligible. In some studies this care has not been taken, and the data have been seriously contaminated by the presence of replacement demand.

For some products there will be multiple users of the adopted product. This brings into question the appropriate definition of the adopting unit. In certain instances such as software, for example, where the product can be copied easily and perhaps illegally, care must be taken to bring the definition of the adopting unit into line with the purpose of the study. Givon et al. (1995) have shown how to estimate the effect of software piracy on lost sales in the diffusion of software.



A. Adoptions Due to External and Internal Influences in the Bass Model.



B. Analytical Structure of the Bass Model.

Source: Mahajan, Muller, and Bass (1990).

FIGURE 1. The Bass New Product Diffusion Model.

2.2. *Category Demand*

The model applies to the generic demand for a product category. However, Krishnan and Bass (1994) have shown that, in at least some instances, the model will apply to the demand for individual brands and to niches within a category, as well as to the category adoption pattern.

2.3. *Supply Restrictions*

Demand growth for a new product can be retarded by supply restrictions because of limited production capacity or difficulties in establishing distribution systems. Jain et al. (1991) have used telephone data from Israel to show how capacity decisions can be linked to demand growth.

3. **The Empirical Generalizations**

3.1. *The Pattern of Adoption*

More than 150 papers have been written based on refinements, extensions, and applications of the Bass model (see Mahajan et al. 1990 for a review and classification of these studies). Although there are exceptions, the model usually provides a good fit to the adoption data, and the adoption pattern normally looks like the lower left panel of Figure 1. Jeuland (1994) has fitted the model to 35 data sets for different time periods

and different countries and finds that the model always provides a good fit to the data with  $R^2$  values greater than 0.9. The Bass model itself, then, constitutes an empirical generalization and the pattern shown in 1B and mathematical summarizations of the generalization.

### 3.2. Generalizations Concerning $p$ and $q$

In addition to scholarly studies, there have been hundreds of real-world applications of the model. These applications have consisted primarily of forecasts of demand prior to introduction of the product and before data were available to estimate the parameters of the model. Guessing algorithms, some of them computerized, have been developed for application purposes. In application, a major interest is upon the timing of the peak in adoption and the peak, of course, depends upon the parameters  $p$  and  $q$ . Lawrence and Lawton (1981) have found that the value of  $p + q$  lies between 0.3 and 0.7. Sultan et al. (1990) analyzed the parameter estimates of 213 published applications of the Bass model and its extensions. They report the average value of  $p = 0.03$  and the average value of  $q = 0.38$ . Jeuland (1993 and 1994) finds that the value of  $p$  is often quite small, 0.01 or less. On the other hand,  $q$  is rarely greater than 0.5 and rarely less than 0.3. The distribution based on historical data as well as the range and average values of the parameters provides useful generalizations concerning parameters for purposes of application when no data are available for estimation, in that they provide benchmarks for comparing guesses with unlikely values based on historical estimates.

### 3.3. Adopter Categories

Rogers (1983) has hypothesized that the adoption pattern is a normal distribution. Using the number of standard deviations away from the mean of the normal distribution, Rogers has suggested that there are five categories of adopters based on when adoption occurs. The categories and percentage of adopters in each category are shown in Figure 2. Mahajan et al. (1990) have explicated that since one standard deviation on either side of the mean of the normal distribution represents its point of inflection, when the same logic is applied to the Bass model it also generates five adopter categories. These adopter categories are depicted in the lower panel of Figure 2. As compared to the Rogers' classification that assumes that the percentage of adopters for the five categories is invariant across innovations, the Bass classification is innovation specific. Hence for the Bass model the category time interval and the percentage of adopters in each of the five categories vary across innovations and depend upon  $(p + q)$  and  $q/p$ . The range of percentages for each adopter category shown in the lower panel of Figure 2 is alternative way of expressing the range of parameter values over many different innovations.

### 3.4. Successive Generations of Technology

Norton and Bass (1987, 1992) modified the Bass model to apply to successive generations of technology. Figure 3 shows the actual and fitted model for sales of four generations of DRAMS (dynamic random access memory products). It is clear from the graph that the fits are quite good. The intergenerational growth and substitution model is:

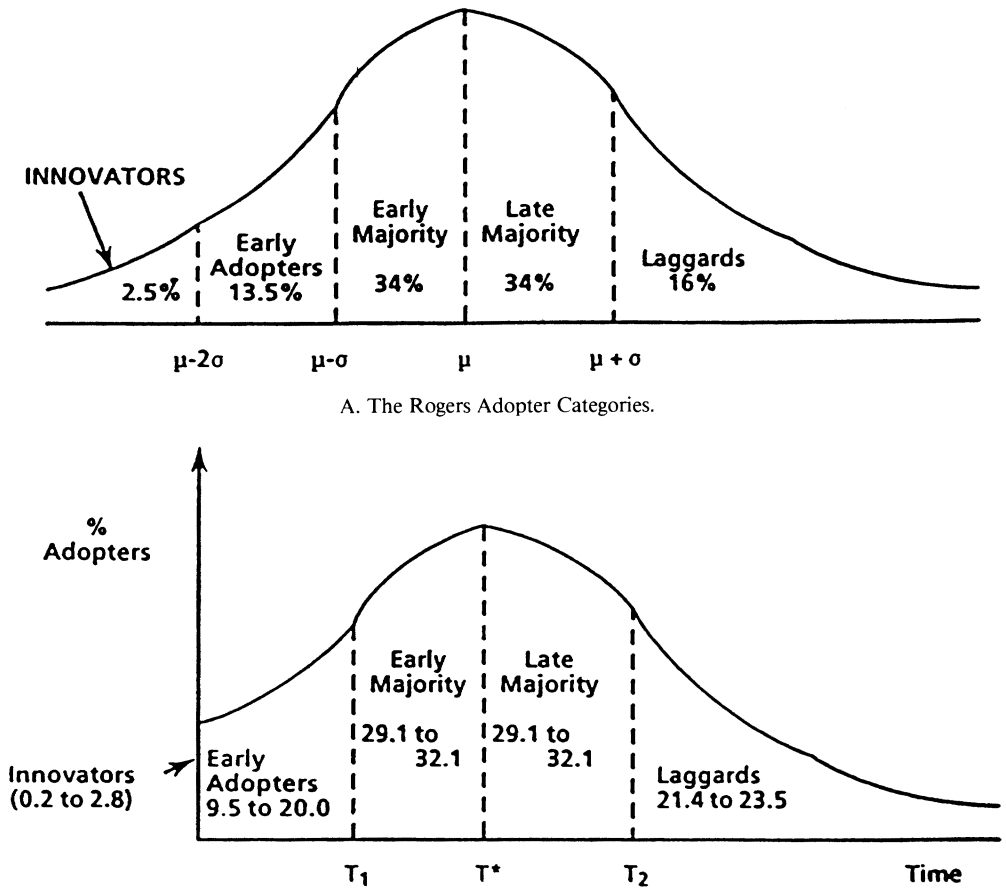
$$S_{1,t} = F(t_1)m_1[1 - F(t_2)], \quad (4)$$

$$S_{2,t} = F(t_2)[m_2 + F(t_1)m_1][1 - F(t_3)], \quad (5)$$

$$S_{3,t} = F(t_3)[m_3 + F(t_2)[m_2 + F(t_1)m_1]][1 - F(t_4)], \quad (6)$$

$$S_{4,t} = F(t_4)[m_4 + F(t_3)[m_3 + F(t_2)[m_2 + F(t_1)m_1]]], \quad (7)$$

where  $m_i$  is the incremental market potential for the  $i$ th generation,  $t_i$  is the time since



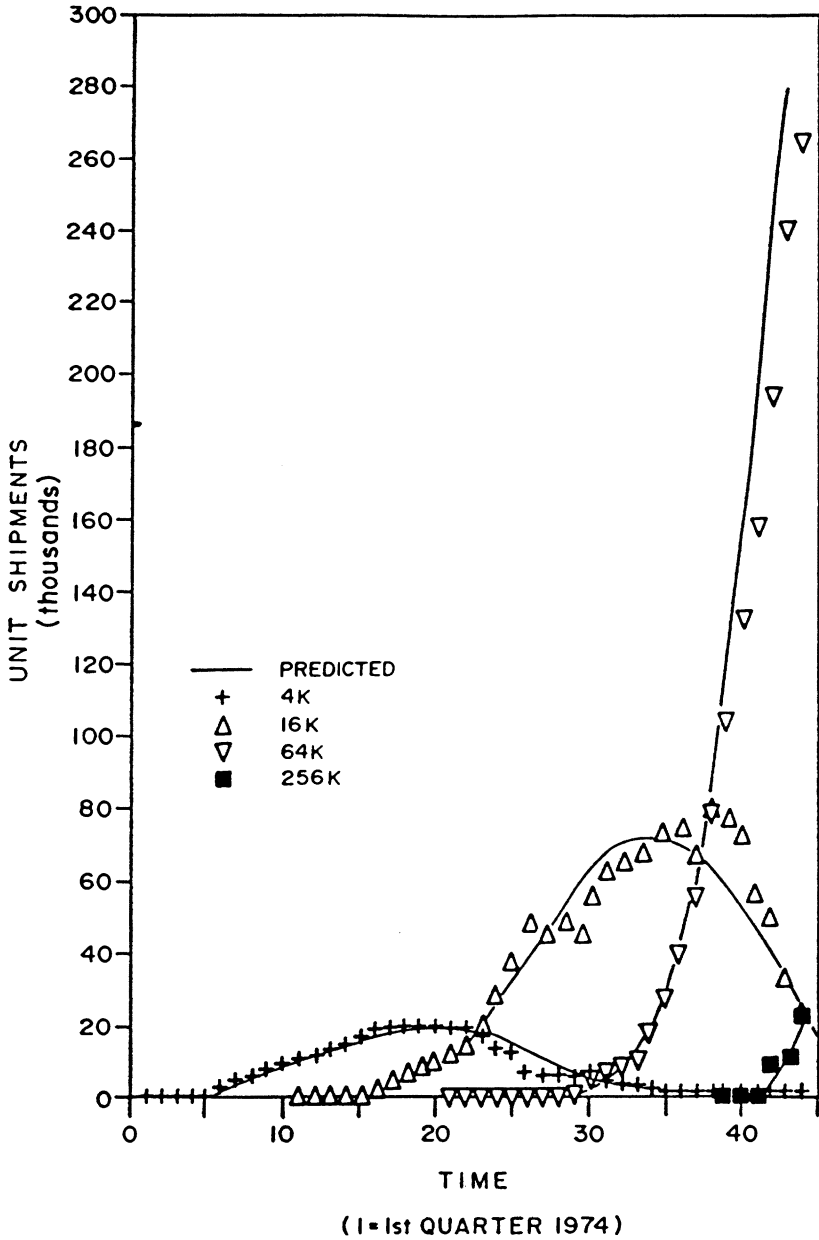
B. The Bass Adopter Categories.\*

\* The percentage range reported for each category is based on the parameter estimates reported by Bass (1969).

Source: Mahajan, Muller, and Srivastava (1990).

FIGURE 2. A Comparison Between the Bass Model and Classical Normal Distribution Model.

the introduction of the  $i$ th generation, and  $F(t_i)$  is the cumulative distribution associated with the solution to the differential equation for the Bass model. Parameter estimation for  $p$ ,  $q$ , and  $m_i$  involves nonlinear estimation of the system of four simultaneous nonlinear equations. Norton and Bass (1992) estimated the parameters and examined the fits of this model for 12 product categories for products in electronics, pharmaceuticals, consumer products, and industrial product categories and found that in every instance the fits were extremely good and that the demand patterns were similar to that shown in Figure 3. This model is derived from the Bass model and implies that the underlying diffusion process of this model governs the process of adoption and disadoption for successive generations of technologies. The derived model, then, is an empirical generalization in its own right. Moreover, Norton and Bass found that, in almost every instance, the values of  $p$  and  $q$  were unchanging over different generations. This empirical generalization has important implications from an applied forecasting standpoint in that it is possible to estimate parameters based on data from early generations to forecast diffusion of later generations conditional on the timing of introduction of the later generations.



Source: Norton and Bass (1987).

FIGURE 3. Model Fit to DRAM Data.

### 3.5. Decision Variable Generalizations

Bass et al. (1994) have reviewed 18 models that have modified the Bass model to include decision variables and find that none of the prior models reduce to the Bass model as a special case except when decision variables are constant. Thus none of the previous models can explain why the Bass model will describe adoption data. A generalized version of the model that includes decision variables such as price and advertising has been developed that under plausible and observable behavior of change in decision variables reduces to the Bass model. This generalized model has been termed the Generalized

Bass Model (GBM), and with this model the diffusion curve may be shifted in time, but the shape maintained, with different sequences of the decision variables. When decision variables (price, for example) change by a constant percentage, the Generalized Bass Model will reduce to the Bass model. An often observed empirical generalization concerning price for new products and technologies is that prices fall exponentially in time. Exponential fall in price is consistent with a constant percentage decline in price. Moreover, it can be shown that, under myopic optimization, when marginal costs follow the experience curve form, a constant percentage reduction in price will be approximately optimal. A similar result can be shown for percentage increases in advertising up to the time of peak demand. Empirical regularity concerning the behavior of decision variables, then, can be used to explain why the Bass model almost always fits the data even though it does not include decision variables.

#### 4. Applications

Although, as previously indicated, most real managerial applications of the model have been in the arena of forecasting prior to product introduction, the potential exists for normative and strategic applications as well. We shall briefly discuss here issues in forecasting and some possible strategic and normative applications.

##### 4.1. *Forecasting*

Forecasting adoption timing prior to product introduction will obviously require guesses of parameters. At least two procedures for guessing have been suggested. An algebraic estimation procedure has been suggested by Mahajan and Sharma (1986) that requires expert judgments to obtain three pieces of information about the adoption curve: (1) market size, (2) the time of the peak in the adoption rate, and (3) the adoption rate at the peak. Lawrence and Lawton (1981) have suggested a procedure that is based on guesses of  $m$ ,  $q/p$ , and  $p + q$ . Experience indicates that for annual data  $p + q$  varies between 0.3 and 0.7. Judgments are obtained from managers as to whether the product is likely to be: (a) slightly contagious, (b) moderately contagious, or (c) highly contagious, and the value of  $p + q$  is calibrated on these judgments by selecting appropriate values within the interval (0.3, 0.7). Judgments are also obtained from managers about likely first year sales and market potential. Given  $p + q$  and first year sales, the ratio  $q/p$  may be determined algebraically. Because the parameter  $p$  is usually quite small and estimates of it unstable (see Jeuland 1994), "guessing by analogy" is more likely to be successful when it is based on  $q/p$  and  $p + q$  than on  $p$  and  $q$ . Bayus (1993) has developed a forecast of high-definition television (HDTV) based upon a grouping of analogous products for which historical data are available. He has concluded that among three forecasts, one by the American Electronic Association (AEA), one by the Electronics Industries Association (EIA), and one by the National Telecommunications Information Administration (NTIA), only the forecast by AEA is consistent with historical data for analogous products. These results, and others like them, suggest that even when the forecasts based on the model are not very accurate, the model may be used to determine whether or not non-model-based forecasts imply parameter values that fall outside the "feasibility space" suggested by historical norms.

##### 4.2. *Normative Effects of Sampling*

For certain products, such as pharmaceuticals, software, and textbooks, product sampling may be an effective means for introducing a new product and stimulating word-of-mouth effects. When sampling is to be employed, key questions concern the number of samples to distribute and to whom. Jain et al. (1995) have examined this issue and have concluded that target sampling to opinion leaders and innovators is more effective

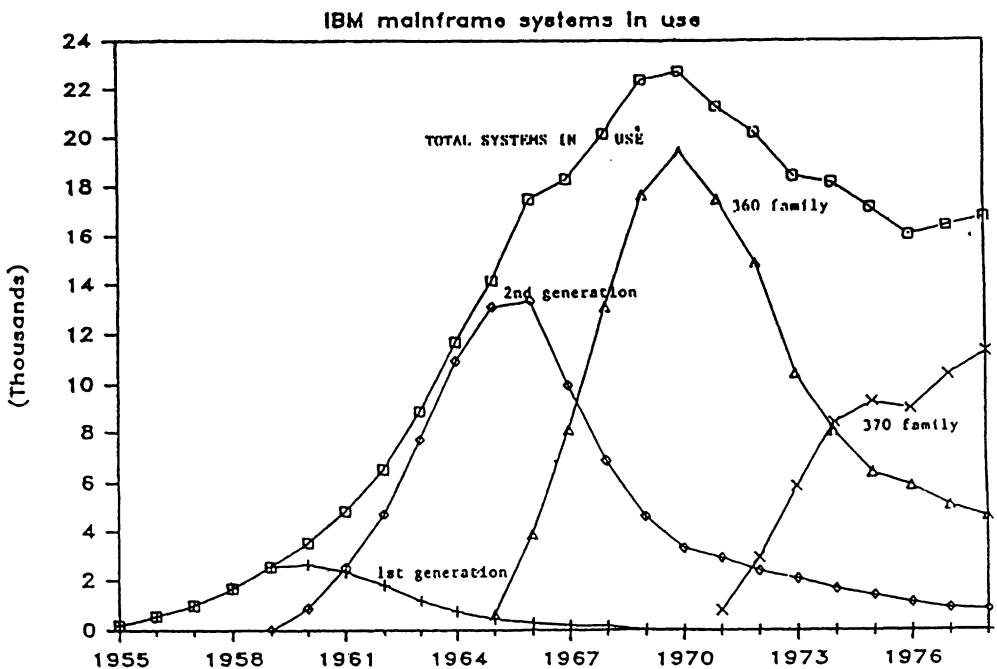
than neutral sampling, and they further suggest that sampling for durables should be no more than nine percent of the total number of potential adoptions.

#### 4.3. Normative-Introduction Timing for Successive Generations

Although a successive generation of technology may produce an increment in potential buyers in comparison with earlier generations, the later generation's entry timing will influence both its own diffusion and the diffusion rate of earlier generations. Wilson and Norton (1989) were the first to study the issue of optimal introduction timing when product generations are involved. More recently, Mahajan and Muller (1995) have studied the introduction timing issue and have suggested that the optimal introduction timing will depend on the relative size of the market potential of the various generations, gross profit margins, the diffusion and substitution parameter values of earlier generations, and the cost of capital for the firm. They have studied the demand patterns for successive generations of mainframe IBM computers from 1955 through 1978 and have concluded that IBM introduced the two successive generations of 360 and 370 families too late. Figure 4 shows that the demand patterns for IBM mainframe generations follow a similar pattern to that shown in Figure 3 based on the Norton and Bass model.

#### 4.4. Normative-Damage Assessment in Patent Infringement

In 1976 Kodak introduced an instant camera to compete with Polaroid, which had invented instant photography, and which prior to that time had enjoyed a monopoly in the instant camera business. Polaroid sued Kodak for patent infringement and, following a long legal struggle, was awarded \$909.5 million in damages in October 1990, representing the largest award for patent infringement in history. Using a model similar to the Norton and Bass model, Mahajan et al. (1993) have analyzed data from the instant camera



Source: Mahajan and Muller (1995).

FIGURE 4. Diffusion and Substitution.



market between 1963 and 1988 and have concluded that Kodak's entry expanded the U.S. instant camera market by 37 percent, but from 1977 to 1985 Kodak drew about a third of its buyers from Polaroid.

#### 4.5. Normative Value of a Business

In growth markets such as cellular communications where the adoption rate is accelerating and where mergers and acquisitions are increasing, it is possible to link the value of a business with diffusion rates and the price paid for the business. Kim et al. (1995) have shown that the price per pop (price paid for a cellular telephone company per capita in its trading area) increases with the penetration rate for cellular telephones and have developed a model that links market penetration and price per pop.

### 5. Conclusion

The Bass diffusion model constitutes an empirical generalization. In addition, other empirical generalizations have been spawned by applications and extensions of the model. Early applications of the model were restricted to forecasts, but as we have shown here, a variety of other applications areas have emerged. We expect further empirical generalizations to emerge around existing and future application areas.

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