VIJAY MAHAJAN & EITAN MULLER

This paper assesses the state-of-the-art of the diffusion models of new product acceptance. A number of issues related to the further development and validation of these models are discussed.

INNOVATION DIFFUSION AND NEW PRODUCT GROWTH MODELS IN MARKETING

THE recent literature in new product planning has been concerned with developing better approaches for: (1) the generation and testing of new product concepts (Green and Wind 1975; Pekelman and Sen 1974; Shocker and Srinivasan 1974; Silk 1969; von Hippel 1978), (2) pretest market evaluation of new products (Claycamp and Liddy 1969; Silk and Urban 1978; Tauber 1977), (3) test-market evaluation of new products (Assmus 1975; Blattberg and Golanty 1978; Learner 1968; Urban 1970), (4) product-line strategy (Day 1977; Wind and Claycamp 1976), (5) product life cycle strategy (Dhalla and Yuspeh 1976), and diffusion models of new product acceptance (Mahajan and Peterson 1979b). This paper is concerned with diffusion models of new product acceptance.

The objective of a diffusion model is to represent the level of spread of an innovation among a given set of prospective adopters in terms of a simple mathematical function of time that has elapsed since the introduction of the innovation. The purpose of the model is to depict the successive increase in the number of adopters and predict the continued development of a diffusion process already in progress. In the product-innovation context, diffusion models are concerned with the spread of a new product from its manufacturers to ultimate users or adopters and focus on the development of a product life cycle curve (Kotler 1971; Wind 1974). The underlying behavioral theory in the development of these models is that new product acceptance is an imitation process (Rogers and Shoemaker 1971).

Vijay Mahajan is Assistant Professor of Marketing, College of Administrative Science, Ohio State University, Columbus, OH. Eitan Muller is Assistant Professor of Economics, University of Pennsylvania, Philadelphia, PA, and Lecturer, School of Business Administration, The Hebrew University, Jerusalem, Israel.

In recent years, a number of models have been developed to represent the spread of a new product in the marketplace. Most of these have their roots and analogies in the models of epidemics, or biology and ecology (Bailey 1957; Pearl 1925) and serve the purpose of forecasting sales for durable goods and novelty items. The objective of this paper is to review and assess the state-of-the-art of these modeling efforts. It is our hope that this review would provide the marketing managers and researchers with a simple and systematic overview of the development of diffusion models of new product acceptance. In addition, we hope to provide answers to such questions as: What are the underlying assumptions of these models? Where have these models been applied? How are the proposed models different from each other? What are the shortcomings of these models? What directions need to be followed to make these models theoretically more sound and practically more effective and realistic? The marketing tradition of diffusion research has come on strong since the early 1960s (Rogers 1976). However, most of the diffusion models of new product acceptance have been developed in the

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last decade or so. A fair amount of the diffusion research in marketing has been and is being conducted in industry (e.g., Eastman Kodak, Mitre Corporation). Our presentation is restricted to the review and evaluation of the published literature.[1] Furthermore, the presentation is generally related to first-purchase diffusion models with some discussion of repeat purchase models.

The format of this exposition is as follows. First, the general underlying structure employed in the development of diffusion models and the notation used in the paper are presented. Next, diffusion models are reviewed. Finally, issues related to the further development, validation, and evaluation of these models are discussed.

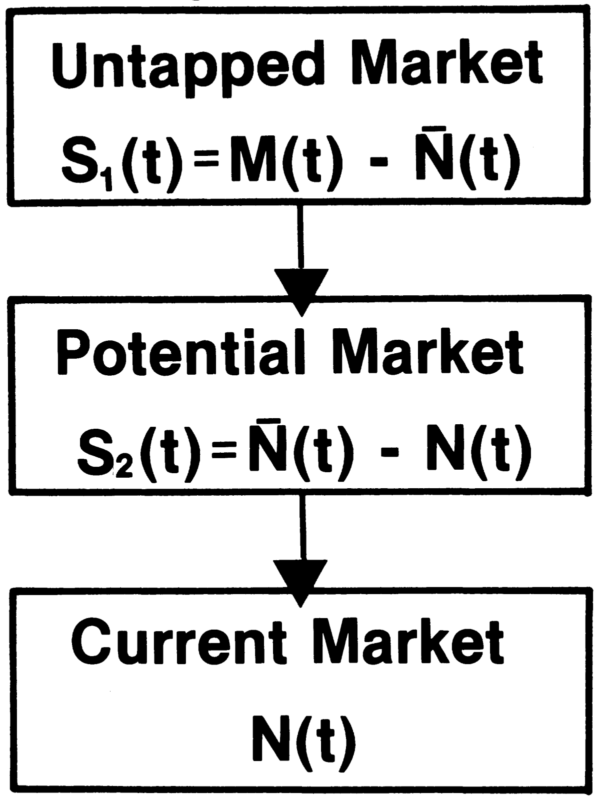
A General Diffusion Model Structure

The first-purchase diffusion models of new product acceptance assume that, in the product planning horizon being considered, there are no repeat buyers and purchase volume per buyer is one unit. In the development of these models, a model builder, in general, is concerned with the following question:

> At any time t in the diffusion process, if there are total $M(t)$ number of individuals in the market, and $\bar{N}(t)$ number of them can be considered as the potential customers (or the eventual number of buyers), how many of the potential customers will have bought the product by time t?

In other words, the analyst is concerned with (1) the total flow of customers and (2) the rate of flow of customers, across three distinct segments of the market. The first segment, the *untapped market*, consists of $(M(t)-\bar{N}(t))$ number of individuals. These are the individuals who are either uninformed about the product or due to a number of factors (such as high price of the product, general economic conditions, etc.) cannot be considered as the potential customers at time t. However, over time, individuals from this segment may transfer to the next segment, the *potential market*, consisting of $(\bar{N}(t)-N(t))$ number of potential customers at time t. The last segment, the *current market*, of course, includes the $N(t)$ number of adopters who have bought the product by time t. The summation of the number of the individuals in each segment totals the market, i.e.,

FIGURE 1
Customer Flow Across the Three Market Segments in the Diffusion Process



$$S_1(t)+S_2(t)+N(t)=M(t) \tag{1}$$

where,

$$S_1(t)=M(t)-\bar{N}(t) \tag{2}$$

$$S_2(t)=\bar{N}(t)-N(t) \tag{3}$$

where $S_1(t)$ and $S_2(t)$ represent the number of customers in the first two segments, untapped market and potential market, respectively. Equations (1), (2), and (3) characterize the customer flow in the diffusion process. This flow is summarized in Figure 1. Furthermore, between time period t and $t+1$, if $n(t)$ number of individuals transfer from the potential market segment to the current market segment, $\bar{n}(t)$ number of individuals transfer from the untapped market to the potential market and the total market increases by $m(t)$ (i.e., $m(t)=M(t+1)-M(t)$), then at time $(t+1)$ the number of individuals in the three market segments are given by the following equations:

$$S_1(t+1)=S_1(t)+m(t)-\bar{n}(t) \tag{4}$$

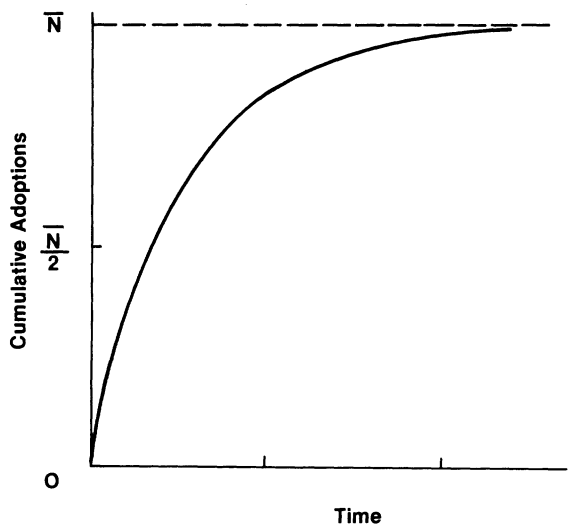
$$S_2(t+1)=S_2(t)+\bar{n}(t)-n(t) \tag{5}$$

$$N(t+1)=N(t)+n(t) \tag{6}$$

Equation (4) implies that number of individuals

[1]There are at least 18 different disciplines which study the diffusion of innovation looking at its different aspects and different contexts. The review in this paper is restricted to marketing literature with some key borrowings from economics literature. Although Rogers and Shoemaker (1971) and Robertson (1971) provide excellent reviews of the state-of-the-art until 1971, some recent reviews, for example by Brown (1979) in geography, Hurter and Rubenstein (1978) in technological substitution, and Rogers (1976) in communication, are very informative. The spatial diffusion models of new product acceptance also are not reviewed. Brown (1979) and Mahajan and Peterson (1979a) provide a review of these modeling efforts.

FIGURE 2
Modified Exponential Function Diffusion Process



in the untapped market at time $(t+1)$ is equal to number of individuals at time t plus the increase in the number of individuals in the total market minus the number of individuals who transfer from the untapped market to the potential market. Equation (5) suggests that the population of the potential market at time $(t+1)$ is increased by the number of individuals who transfer from the untapped market to the potential market and decreased by the number of individuals who transfer from the potential market to the current market. Equation (6) reflects the increase in the number of adopters due to the $n(t)$ number of individuals who transfer from the potential market to the current market. If $s_1(t)=S_1(t+1)-S_1(t)$ and $s_2(t)=S_2(t+1)-S_2(t)$, equations (4) and (5) may be written as

$$s_1(t)=m(t)-\bar{n}(t) \tag{7}$$

$$s_2(t)=\bar{n}(t)-n(t) \tag{8}$$

Furthermore,

$$s_1(t)+s_2(t)+n(t)=m(t) \tag{9}$$

Equations (7), (8), and (9) represent the rate of the flow of customers across the three segments of the diffusion process. In fact, the customer flow equations, (1)-(3), and the rate equations, (7)-(9), form the basis for all the proposed diffusion models. The development of these models basically involves the representation of the rate of customer flow across the segments in the diffusion process in terms of the transfer mechanisms. In general, five transfer mechanisms, which effect the flow of individuals from one segment to the next, may be considered. These are: (1) mass-media communication, (2) word-of-mouth communication, (3) other marketing efforts, (4) individual experience with the product, and (5) exogeneous factors (e.g., general economic environment).

The available diffusion models differ in terms of the specific segments and the transfer mechanisms considered in their development. The following section provides a detailed comparison of these models.

Product Growth Models

The best-known, first-purchase diffusion models of new product acceptance in marketing are those of Bass (1969), Fourt and Woodlock (1960), and Mansfield (1961). The Bass model has been successfully demonstrated in retail service, industrial technology, agriculture, and consumer durable sectors (Bass 1969; Dodds 1973; Nevers 1972), while the Fourt and Woodlock model has been used to study success of certain grocery products (Fourt and Woodlock 1960). The Mansfield model and its revised forms, such as those proposed by Blackman (1974) and Fisher and Pry (1971), have been used in technological substitution studies of industrial innovations.[2]

The Fourt and Woodlock model depicts the diffusion process, in terms of the number of customers who have bought the product by time t, (i.e., $N(t)$), by a modified exponential curve (see Figure 2). The Mansfield model, on the other hand, represents the diffusion process by a logistic curve (see Figure 3). The Bass model synthesizes these two approaches and employs a generalized logistic curve which contains the Fourt and Woodlock, and Mansfield models as its special cases. Related to the logistic curve is the Gompertz curve which also has been used to model the product growth (e.g., Hendry 1972) (see Figure 4). The modified exponential curve, logistic curve, and Gompertz curve are the *basic* diffusion models of product growth.

In modeling the growth of the first-time buyers of a product, the basic diffusion models consider only two segments in the diffusion process—potential market and current market—and two transfer mechanisms to influence the potential customers to adopt the product—mass-media communication and word-of-mouth communication. Furthermore, these models assume a *constant* total population of potential customers over the entire life of the product, i.e., $\bar{N}(t)$ is constant or

[2]The technological forecasting literature identifies the diffusion process mainly as a substitution process by which the adoption of a new product or technology spreads and grows to replace an existing product or technology. Linstone and Sahal (1976) provide an excellent review of this literature.

FIGURE 3
Logistic Function Diffusion Process

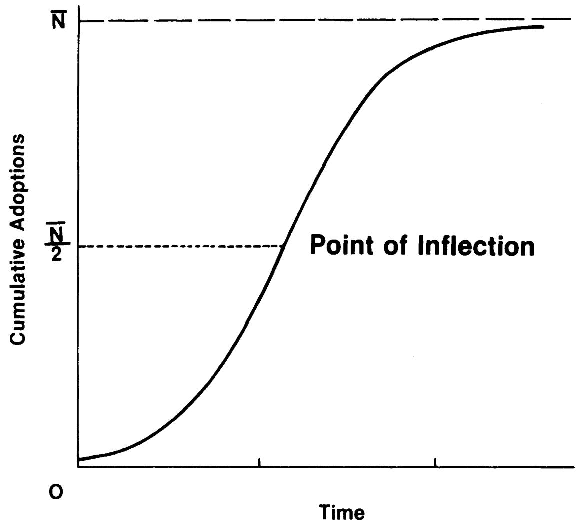
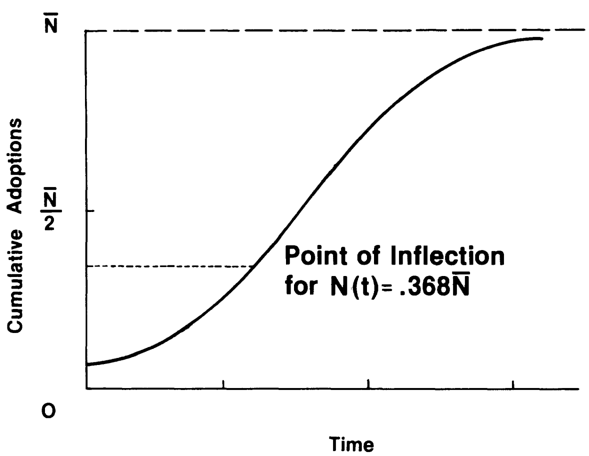


FIGURE 4
Gompertz's Function Diffusion Process



$\bar{n}(t)=0$. In other words, basic models are concerned with the following question:

> If there are $\bar{N}$ number of potential customers in the market, how many of the potential customers will have bought the product by time t?

These models are *primarily concerned with modeling* $n(t)$, the flow of customers from the potential market to the current market.

A number of efforts have been made to extend these basic diffusion models. The extensions proposed by Bass (1978), Horsky and Simon (1978), Lilien and Rao (1978), Peterson and Mahajan (1978), and Robinson and Lakhani (1975) have been concerned with the inclusion of the effects of other transfer mechanisms in the basic diffusion models. However, these models also consider the customer flow across two segments of the diffusion process only—the potential market and current market—and assume a constant total population of potential customers, i.e., $\bar{N}(t)$ is constant. Hence, these extensions also have been *primarily concerned with modeling* $n(t)$. On the other hand, extensions proposed by Chow (1967), Dodson and Muller (1978), Lackman (1978), and Mahajan and Peterson (1978) consider the customer flow across all three segments of the diffusion process. Furthermore, these extensions implicitly consider the following question:

> If there are M number of individuals in the market and $\bar{N}(t)$ number of them can be considered as potential customers, how many of the potential customers will have bought the product by time (t)?

Note in the above question that these models assume that (1) M, number of individuals in the market, is constant, i.e., $m(t)=0$; (2) $\bar{N}(t)$, total number of potential customers in the market, changes over time. In other words, these models are concerned with *modeling* $n(t)$*, the flow of customers from the potential market to the current market, as well as* $\bar{n}(t)$ *(or* $\bar{N}(t)$*), the flow of customers from the untapped market to the current market.* Finally, in extending the basic diffusion models to include the third segment, untapped market, Mahajan et al. (1979) relax the assumption that $M(t)$—the number of individuals in the market—is constant. In addition to modeling $n(t)$, $\bar{n}(t)$, they also are concerned with modeling the growth of the market, $m(t)$. The customer flow and the rate of customer flow equations for these models are given in Appendix A.[3]

Basic Product Growth Models

Considering the rate of customer flow between two segments of the diffusion process, the potential market and the current market, Bass (1969) proposed the following growth model for durables:

$$n(t)=p(\bar{N}-N(t))+\frac{q}{\bar{N}}(\bar{N}-N(t))N(t) \qquad (10)$$

In the development of this model, Bass considered the underlying theory of innovative behavior which states that the diffusion of innovation is a communication process. In fact, he employed the so-called two-step flow of communication model (Robertson 1971). According to this model, the message about the

[3]Some further possible extensions of the basic models are discussed in Bernhardt and MacKenzie (1972). In the communication literature, Rogers and Shoemaker (1971) have suggested that the noncumulative adoption curve is a normal distribution. However, the cumulative normal distribution also yields a logistic curve, one of three basic models presented here.

product (through the mass media) is first picked up by a select few innovators who then "pass the word" to other members of the social system. Bass identified the constant p as the coefficient of innovation since the term containing the constant p in equation (10) represents adoptions without "interaction" with adopters. Similarly, he termed the constant q as the coefficient of imitation to reflect the word-of-mouth communication between adopters, $N(t)$, and potential adopters, $(\bar{N}\text{-}N(t))$. He further illustrated that for a successful product, the value of q should be greater than the value of p.

The interesting feature of the Bass model, equation (10), is that it unified the two earlier models proposed by Fourt and Woodlock (1960) and Mansfield (1961). Fourt and Woodlock demonstrated that the growth of certain grocery products can be modeled as the modified exponential curve, i.e.,

$$n(t) = p(\bar{N} - N(t)) \tag{11}$$

Mansfield, on the other hand, found that the growth of certain technological innovations can be represented by an S-shaped or logistic curve, i.e.,

$$n(t) = bN(t)\ (\bar{N} - N(t)) \tag{12}$$

Note $b = \frac{q}{\bar{N}}$ in the Bass model. Further note that the Fourt and Woodlock model, equation (11), assumes diffusion only through innovators and the Mansfield model, equation (12), assumes a pure imitation diffusion process. In a recent paper, Lekvall and Wahlbin (1973) further elaborate on the interpretation of these models. They question the validity of the two-step flow-of-communication model and suggest that in the communication process, each potential adopter is subjected to two different forms of influence—external and internal. External influence is the direct influence on the innovative behavior of an individual which, for example, the marketer of a new product exerts through various promotional activities such as mass media advertising, public exposure of the new product, or personal discussion with prospective adopters by professional salespeople. Internal influence, on the other hand, is the influence that the members of a social-system exert on one another as a result of social interaction. Furthermore, these two influences may be operating simultaneously on an individual at any time in the diffusion process. Using these two forms of the influence, Lekvall and Wahlbin suggest the following growth model:

$$n(t) = \frac{p}{\bar{N}}(\bar{N} - N(t)) + \frac{q}{\bar{N}}N(t)\ (\bar{N} - N(t)) \tag{13}$$

Note that the formulation suggested by equation (13) is exactly like the formulation suggested by equation (10), except that the constant p has been divided by $\bar{N}$. Unlike Bass, however, Lekvall and Wahlbin interpret the constant p as the coefficient of external influence representing the influence of promotional activities of a company. The constant q still represents the internal influence or the imitation effect. Commenting further on these two coefficients, Lekvall and Wahlbin write:

> In most real world situations external and internal influence will interact to produce a certain outcome of a diffusion process. However, the relative strength of these two factors are not likely to be the same in all situations. In some cases the dominating influential power may emanate from sources outside the set of (prospective) adopters, whereas in other situations the influence that the members of this set exert on each other may be more important. Furthermore, theoretical considerations as well as empirical evidence indicate that the nature of the innovation is a crucial factor underlying such differences. For example, whereas a new kind of fertilizer may be subject to much debate among farmers in a community—and thus show a relatively high intensity of internal influence—this is not likely to be the case for a new kind of orange juice or a new brand of household soap. Likewise, whereas the buyer of a new car model will probably initiate substantial internal influence just by using the car, the adoption of disposable toothbrushes, for example, is not likely to yield any considerable demonstration effect.

Through a simulation process Lekvall and Wahlbin generate different shapes of diffusion curves reflecting different intensity levels of p and q.

Following Lekvall and Wahlbin's interpretation of the constants p and q, it is clear that the modified exponential curve or the Fourt and Woodlock model, equation (11), represents the growth of a product which is mainly caused by the external influence or the promotional activities of a company. The logistic curve, equation (12), on the other hand, represents the growth of a product which is mainly caused by the internal influence.

Related to the logistic curve is the Gompertz curve which has also been used to model the diffusion process:

$$n(t) = bN(t)\ (1n\bar{N} - 1nN(t)) \tag{14}$$

As evident from equation (14), like the logistic curve, the Gompertz curve considers only internal influence. However, unlike the logistic curve, the Gompertz curve expresses the difference between the total number of individuals in the potential market and current number of adopters in terms of their natural logarithms. A further observation on the difference between these two growth curves is in terms of the stage at which the maximum rate of growth, $n(t)$, is reached. Setting the derivative of equation (12) with respect to $N(t)$ equal to zero, one finds that $N(t) = .5\bar{N}$. Setting the derivative of equation (14) equal to zero, one finds that $N(t) = e^{-1}\bar{N} = .37\bar{N}$. Hence, the maximum rate of growth is reached when the total number of adopters is about 37% of its potential market by the Gompertz curve, and when the total number is 50% of the potential market by the logistic curve. To put it in other words, if the Gompertz curve is used the number of buyers at any time t, $n(t)$, cannot be greater than 37% of the potential market. Furthermore, the logistic curve is symmetrical, i.e., it gives the same value of $n(t)$ *for* $\bar{N}(t) = .5\bar{N} + k$ or $\bar{N}(t) = .5\bar{N} - k$, where k is constant. On the other hand, the Gompertz curve attains its maximum rate of growth at an earlier stage and maintains a more nearly constant rate of growth later on, than does the logistic curve. The application of the Gompertz curve has been demonstrated by Hendry (1972) to model the growth of certain durables in the United Kingdom.

Extensions of the Basic Models of Product Growth

The major criticism of the basic diffusion models of new product acceptance is that they are of little use to the new product manager since they consider diffusion as a function of time only and the marketing program of a company does not enter explicitly as a variable inhibiting the evaluation of the effect of different marketing strategies on the product growth. Since all these growth curves essentially contain three parameters, i.e., coefficient of external influence, coefficient of internal influence, and the total number of potential customers, marketing efforts may be incorporated into these models by representing these parameters as a function of relevant variables, i.e.,

$$a(t) = A(S(t)) \tag{15}$$

$$b(t) = B(S(t)) \tag{16}$$

$$\bar{N}(t) = \bar{N}(S(t)) \tag{17}$$

where $a(t)$ is the coefficient of external influence at time t, $b(t)$ is the coefficient of internal influence at time t, $\bar{N}(t)$ is the total number of potential customers at time t, and $S(t)$ is a vector of all relevant marketing decision varibles. By definition, then, the basic diffusion models of new product growth, i.e., modified exponential curve, logistic curve, generalized logistic, and Gompertz curve, assume that the marketing program of a company remains constant over the entire life of the product growth.[4] The various extensions proposed, however, do not offer a unified theoretical framework to include the effects of marketing variables on the product growth. For this reason, each individual extension will be presented separately. First, consider the extensions to the generalized logistic curve, equation (10) or (13).

(a) Robinson and Lakhani (1975) point out that since for consumer durables, the coefficient of external influence (they call it coefficient of innovation as suggested by Bass 1969) has been found to be very small it is the coefficient of internal influence which should be developed as a function of marketing decision variables. They argue that innovators are only a dominant factor in the marketplace during the short period required to achieve the first several percent of market penetration. Therefore, if the coefficient of internal influence is developed as a function of marketing decision variables such as advertising and price, the diffusion model will enable management to evaluate the effect of a certain marketing program on the growth of the product. In their work, Robinson and Lakhani specifically study the effect of price on the product and assume the following values of the three parameters:

$$\begin{aligned} a &= \text{constant} \\ b(t) &= b_1 \exp(-EP(t)) \\ \bar{N} &= \text{constant} \end{aligned} \tag{18}$$

where b_1 is a constant, E is price elasticity, and $P(t)$ is price at time t.

In a recent paper, incorporating the effect of price in the diffusion models, Bass (1978) argues that as a result of learning, costs and prices for new durables decline over time and consequently affect the growth of the product. It is shown by Mahajan and Peterson (1979b) that Bass assumes the following values of the parameters.

$$\begin{aligned} a(t) &= p \cdot k \cdot (N(t))^{\lambda\alpha} \\ b(t) &= \frac{q}{\bar{N}} \cdot k \cdot (N(t))^{\lambda\alpha} \\ \bar{N} &= \text{constant} \end{aligned} \tag{19}$$

where p and q are coefficients of innovation and imitation, respectively (see equation (10)), k is a constant, α is elasticity of demand, and λ is the learning parameter in the experience curve of production quantity and price. Since price is assumed to be inversely related to the quantity sold, equations (19) can also be expressed as a function of price.

Unlike the Robinson and Lakhani model, equations (18), the new Bass model, suggests that both the coefficients of external influence and internal influence change over time and become constant only when $N(t) = \bar{N}$. This development is consistent with the observation made by Kotler (1971) that as more units are sold the coefficient a (and possibly b) should

[4]It should be noted here that since the basic diffusion models focus on the development of a product life cycle curve, incorporation of marketing variables into the three parameters of the diffusion model complements the arguments advanced by Dhalla and Yuspeh (1976) that the product life cycle is not given, but can be controlled by the marketing decision variables.

change over time. Bass has tested his new model for six durables.[5]

(b) Incorporating the effect of advertising into the diffusion model, Horsky and Simon (1978) believe that in the context of new products the primary effect of advertising is to be a direct tool for disseminating information about the existence of the new product. Therefore, expression of the coefficient of internal influence as a function of advertising will result in an effect of second order. They suggest that a correct specification of a diffusion model should express the coefficient of external influence as a function of advertising expenditures. In their model application to a new telephonic banking system, Horsky and Simon assume the following values of the parameters:

$$\begin{aligned} a(t) &= a_1 + a_2 A(t) \\ b &= \text{constant} \\ \bar{N} &= \text{constant} \end{aligned} \tag{20}$$

where a_1 and a_2 are constants and $A(t)$ is the level of advertising expenditures.

(c) Modeling the spread of two ethical drugs aimed at a certain class of doctors, Lilien and Rao (1978) consider the effect of personal selling on the product growth. They point out that one of the most important components of the marketing mix employed by pharmaceutical companies is "detailing"—i.e., personal selling by a force of "detailmen" who visit doctors and describe their portfolio of products, provide free samples, literature, etc. For a new product, the impact of detailing is augmented by the word-of-mouth effect that occurs when doctors first prescribing the product find it satisfactory and recommend it to their colleagues. In their model application, Lilien and Rao consider the following values of the parameters in the first-purchase diffusion model:

$$\begin{aligned} a(t) &= a_1 d(t) + a_2 d^2(t) \\ b &= \text{constant} \\ \bar{N} &= \text{constant} \end{aligned} \tag{21}$$

where a_1 and a_2 are constants and $d(t)$ is the level of detailing at time t.

[5]In the development of his model, Bass derives

$$\bar{N}(t) = (K(1-\lambda\alpha))^{\frac{1}{(1-\lambda\alpha)}}$$

where $K = c(\frac{\alpha}{(\alpha-1)}c_1)^{-\alpha}$ and c, c_1 are constants. Since λ and α are assumed to be constant, $\bar{N}(t)$ is also constant. Hence, in the calibration and the application of his model, Bass uses a constant total market potential, i.e. constant value of $\bar{N}(t)$.

(d) Studying the simultaneous growth of related products, Peterson and Mahajan (1978) argue that new products are not introduced into a vacuum nor do they exist in isolation. Other products exist in the market, and these may have an influence—positive or negative—upon the growth of a new product. They specifically consider four product relationships—independent, complementary, contingent, and substitute. They implicitly incorporate the effect of product relationship into the coefficient of internal influence. For example, in the case of two *complementary* products, the parameters for the *first* product are:

$$\begin{aligned} a &= \text{constant} \\ b(t) &= b_1 + c_1 \frac{N_2(t)}{N_1(t)} \\ \bar{N} &= \text{constant} \end{aligned} \tag{22}$$

where b_1 represents the coefficient of internal influence between the potential adopters and adopters of product 1, and c_1 represents the positive influence of the adopters of product 2 on the potential adopters of product 1. $N_2(t)$ and $N_1(t)$ are total number of adopters of products 2 and 1, respectively.

(e) Considering the effect of marketing efforts and exogeneous variables on the product growth, Mahajan and Peterson (1978) suggest that the objective of the marketing efforts of a company is to effect the total number of potential customers, i.e., $\bar{N}(t)$. In fact, they suggest that $\bar{N}(t)$ changes over time and may even be affected by the exogeneous variables such as general economic conditions, changing characteristics of the individuals in the market, technological changes, government actions, etc. The objective of advertising and distribution channels is to increase the awareness and availability of a new product. The effect of this activity should result in an expansion of the total potential market. Their arguments also are supported by Horsky and Simon (1978) who, justifying the development of their model (extension (b) above), suggest that due to increased competition and reduction in production costs as a result of learning, price reductions will place the product within the budgetary limitations of a great number of potential customers, thus expanding the total potential market of the product. Furthermore, they argue that the objective of product modifications and technological changes is to tailor the product to different segments as the interests of those segments become apparent. The effect of this activity is also likely to manifest itself in an expansion of the potential market, $\bar{N}(t)$.

Figure 5 illustrates the dynamic growth model suggested by Mahajan and Peterson (1978, 1979).

The following features of the dynamic model can be noted in this figure:

- Market potential of the new product is increasing, and the market potential curve is distinctly different, at least in the early stage of the product growth, from the product growth curve.
- The difference between the total number of potential adopters, given by the market potential curve, and the actual number of adopters, given by the product growth curve, at any time t decreases with time, and ultimately,
- The market potential curve coincides with the product growth curve.

In their dynamic growth model development, Mahajan and Peterson (1978) consider the following values of the parameters:

$$a = \text{constant}$$
$$b = \text{constant} \tag{23}$$
$$\bar{N}(t) = f(\text{marketing efforts, exogenous factors})$$

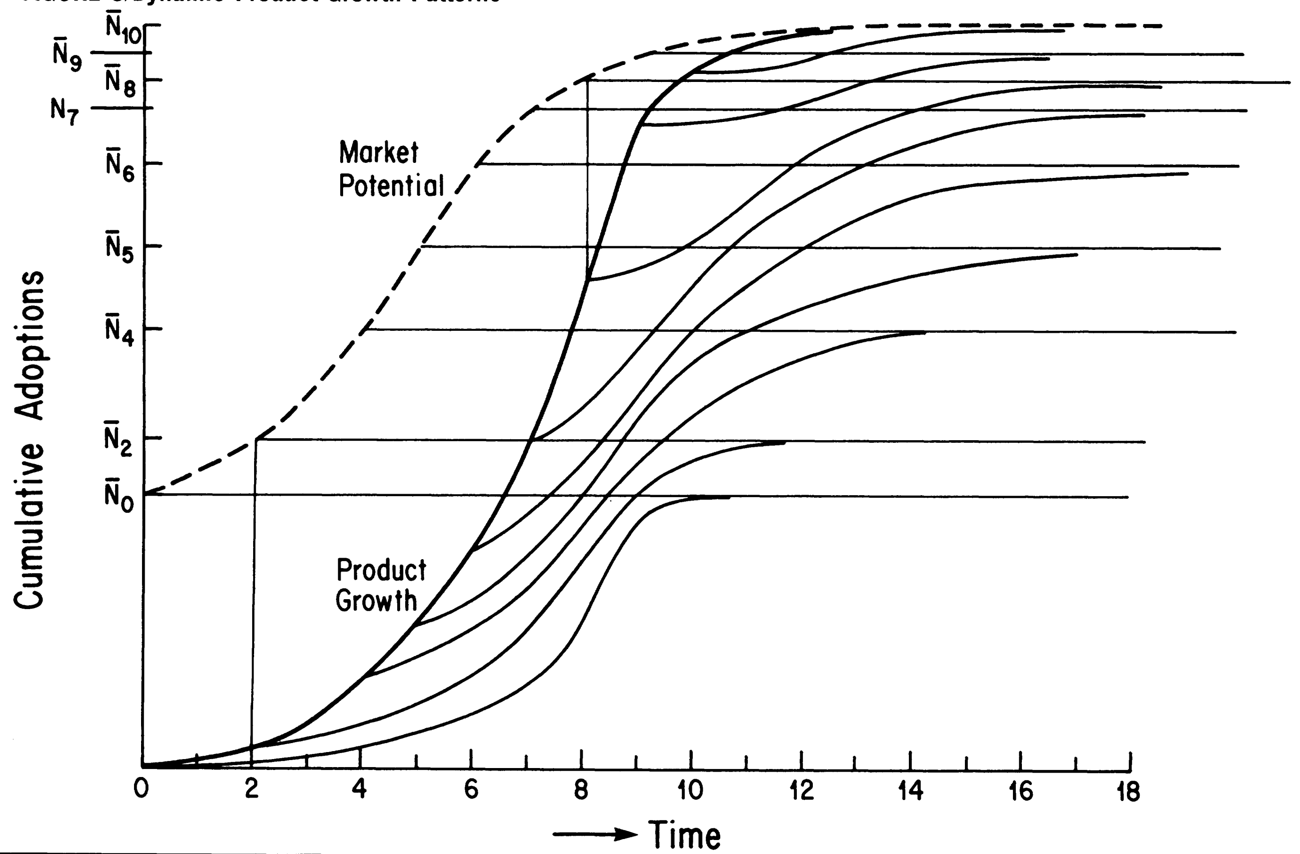
Recalling equations (7)-(9), note here that Mahajan and Peterson consider the flow of customers across all the three segments of the diffusion process. The use of their model involves modeling $n(t)$ as well as $\bar{n}(t)$. In the empirical illustrations of their model for consumer durables, they consider the effect of the construction activities, defined in terms of the number of housing starts, on the product growth (Mahajan et al. 1979). That is, considering that the total market, $M(t)$, changes over time, and recalling equations (7)-(9), they specify the customer flow across the three market segments of the diffusion process by the following equations:

$$m(t) = (h_1 + h_2 M(t))\ (\bar{M} - M(t)) \tag{24a}$$
$$\bar{n}(t) = k_2 m(t) \tag{24b}$$
$$n(t) = bN(t)\ (\bar{N}(t) - N(t)) \tag{24c}$$

where equation (24a) represents the growth of housing starts in terms of the generalized logistic curve (h_1, h_2, and $\bar{M}$ are constants), equation (24b) considers the increase in the number of potential customers (k_2 is a constant), and equation (24c) represents the growth of durables by the logistic curve. From equations (24), it is obvious that they use the following values of the diffusion parameters:

$$a = 0$$
$$b = \text{constant} \tag{25}$$
$$\bar{N}(t) = k_1 + k_2 M(t)$$

FIGURE 5/Dynamic Product Growth Patterns



(f) Incorporating the effect of advertising into the growth models and considering the customer flow across all the three segments of the diffusion process, Dodson and Muller (1978) suggest that since $\bar{N}(t)$, eventual number of buyers at time t, represents the number of potential buyers who are aware of the product, it is $\bar{N}(t)$ which should be expressed as a function of the level of advertising expenditure. In fact, they suggest two mechanisms by means of which unaware individuals may become aware of the product: word-of-mouth communication between product aware and unaware individuals, and advertising. As shown in Appendix B, the Dodson and Muller model is completely specified by the following rate equations:

$$\begin{aligned} \dot{\bar{n}}(t) &= k_1\bar{N}(t)\ (M-\bar{N}(t)) + k_2(M-\bar{N}(t)) \\ n(t) &= a(\bar{N}(t) - N(t)) \end{aligned} \tag{26}$$

Note in equations (26) that Dodson and Muller use the modified exponential curve to model the growth of the product and the logistic curve to model the increase in the total number of potential customers. The constants k_1 and k_2 in equations (26) are interpreted as follows. Since the total number of customers in the potential market, $\bar{N}(t)$, consists of individuals who are aware of the product, $(M-\bar{N}(t))$ represents the individuals who are unaware of the product. Hence the term containing k_2 represents the increase in the number of informed individuals or the potential customers due to external influence such as *advertising*. Similarly, the term containing k_1 represents the increase in the number of informed individuals due to word-of-mouth communication between the informed, $\bar{N}(t)$, and uninformed individuals, $(M-\bar{N}(t))$. Hence, k_1 and k_2 are coefficients of internal influence and external influence, respectively, for the potential market growth. To summarize, Dodson and Muller assume the following values of the diffusion parameters:

$$\begin{aligned} a &= \text{constant} \\ b &= 0 \\ \bar{N}(t) &= f(\text{advertising, word-of-mouth}) \end{aligned} \tag{27}$$

Note here the similarities between the dynamic growth model, equations (23), suggested by Mahajan and Peterson and the model suggested by Dodson and Muller, equations (27). The model suggested by Mahajan and Peterson is a general dynamic growth model and the model suggested by Dodson and Muller presents a mechanism to incorporate the advertising variable into the dynamic growth model.

The above are the seven reported extensions of the generalized logistic curve.[6] Consider, now, the two identified extensions to the Gompertz curve, equation (14). Both of these are related to the inclusion of the effect of marketing efforts and exogeneous factors on the total number of potential buyers, $\bar{N}(t)$. In other words, these extensions consider the flow of customers across all the three segments. However, they assume a constant population of individuals in the market. A brief description of these extensions follows:

(a) Studying the growth of a new plastic product in the automotive industry, Lackman (1978) uses the following parameters:

$$\begin{aligned} b &= \text{constant} \\ \bar{N}(t) &= \bar{N}\cdot\left(\frac{ZB(t)}{SC(t)}\right)^k \end{aligned} \tag{28}$$

where k is a constant and $\frac{ZB(t)}{SC(t)}$ is the ratio of corporate profits to corporate sales in the automotive industry. The profit-sales variable is included to reflect the fact that product users shift to new products quickly when profitability is high.

(b) Examining the natural growth of computers, Chow (1967) argues that the number of computer adoptions have been influenced by the technological changes and price reductions. He implicitly assumes the following formulation to represent this effect:

$$\bar{N}(t) = B_0(P(t))^{-B_1} \tag{29}$$

where B_0 and B_1 are constants and $P(t)$ is the price.

The summary of these extensions is provided in Table 1.

Issues

After a review of underlying assumptions, applications, differences, and shortcomings of various diffusion models of new product growth, it is important to indicate certain key issues which must be addressed in order to make these models theoretically more sound and practically more effective and realistic. These issues relate to the purpose, theory, model formulation, and empirical validation and evaluation of these models.

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[6] Hernes (1976) also has proposed an extension to the generalized logistic curve. He argues that because of the changing characteristics of the population of the social system, and the innovation (technological changes), the coefficients of external and internal influence should change over time. However, he does not explicitly include the effect of exogenous variables on these diffusion parameters. Studying the spread of TV ownership in Norway, Hernes used the following values of the parameters.

$$\begin{aligned} a(t) &= a_1 \cdot a_2^t \\ b(t) &= b_1 \cdot b_2^t \\ \bar{N} &= \text{constant} \end{aligned}$$

where a_1, a_2, b_1, and b_2 are constants. Hernes believes that his model is very flexible in the sense that different types of diffusion curves can be obtained by considering different values of the four constants. For example, if $a_2 = b_2 = 1$, the generalized logistic formulation is obtained.

TABLE 1
First-Purchase Diffusion Models of New Product Acceptance

Work By	Coefficient of Internal Influence	Coefficient of External Influence	Total Number of Potential Customers
Basic Models			
Bass (1969)	constant	constant	constant
Fourt and Woodlock (1960)	0	constant	constant
Mansfield (1961)	constant	0	constant
Gompertz Curve (e.g., Hendry 1972)	constant	0	constant
Lekvall and Wahlbin (1973)	constant	constant	constant
Extensions			
Robinson and Lakhani (1975)	f (price)	constant	constant
Horsky and Simon (1978)	constant	f (advertising)	constant
Lilien and Rao (1978)	constant	f (personal selling)	constant
Bass (1978)	f (demand elasticity, learning parameters, price)	f (demand elasticity, learning parameters, price)	constant
Peterson and Mahajan (1978)	f (product relationships)	constant	constant
Mahajan and Peterson (1978)	constant	constant	f (all relevant variables)
Mahajan et al. (1979)	constant	0	f (housing starts)
Dodson and Muller (1978)	0	constant	f (advertising, word-of-mouth)
Chow (1967)	constant	0	f (price)
Lackman (1978)	constant	0	f (profit/sales)

Purpose. The diffusion models have been developed to estimate new product sales. However, in order to determine the exact shape of the diffusion curve, some early data on the actual diffusion process of the product is required. By the time a new product is launched, investment has been committed and, to that extent, forecasts become less useful. Therefore, for the diffusion models to be useful in developing prelaunch forecasts, various procedures based on laboratory tests and market research need to be examined to estimate model parameters. These approaches may involve the use of parameters derived from an empirical analysis of past new product introductions and experience surveys. The former approach has been illustrated by Dodds (1973) and Lilien and Rao (1978). Using the basic Bass model, equation (10), to study the growth of cable television, Dodds (1973) successfully employed the total market potential estimate, i.e., $\bar{N}$, developed for color television. Similarly, Lilien and Rao (1978) employed the estimates derived from a fully diffused ethical drug to study the spread of a new ethical drug. These approaches seem very promising. The authors understand that Eastman Kodak is working with a number of products to estimate general values for the coefficient of external influence and the coefficient of internal influence in the basic diffusion models.

Included in the parameter estimation issue of these models is the fact that one is working with very few data points and multicollinearity among the variables. One may obtain parameter estimates which are unstable with high standard errors. This may require consideration of approaches such as ridge regression to handle multicollinearity (Mahajan, Jain, and Bergier 1977) or the feedback/adaptive approaches which automatically adjust and update the initial parameter estimates to changing data patterns (Mahajan, Bretschneider, and Bradford 1980). Furthermore, the use of feedback/adaptive approaches may also assist in dealing with the range of the values of the decision variables not encountered in the earlier data.

Theory. From the review of the extensions to the basic diffusion models, it is apparent that each individual effort has tried to incorporate only one relevant variable into the model; and different arguments have been made to incorporate the same variable either in the coefficient of internal influence, coefficient of external influence, or the total number of potential customers. For example, consider the variable advertising. Horsky and Simon (1978) believe that it should be included in the coefficient of external influence. Dodson and Muller (1978) suggest that its effect should be included in the total market potential. Similar is the case with price. Robinson and Lakhani (1975) consider its effect on the coefficient of internal influence. Mahajan and Peterson (1978), Horsky and Simon (1978), and Chow (1967) have argued that its effect should be considered in the total market potential. In spite of all these extensions, a unified theory to incorporate the marketing variables (as well as exogenous factors) into diffusion models is not apparent. Furthermore, these extensions have dealt only with price and promotion. Some work is underway in geography by Brown (1979) to examine the effect of the distribution network on the diffusion process. There remains a woeful lack of accomplishment in this domain.

Model Formulation. Although some significant contributions have been made to the basic diffusion models, a number of further refinements still remain. First is the inclusion of repeat or replacement sales. Of all the models summarized in Table 1,[7] only the models proposed by Dodson and Muller (1978) and Lilien and Rao (1978) have considered this issue.[8] However, both the models are restricted to the first-repeat sales. Furthermore, none of the models has included impulse purchase.

The most important refinement, however, is related to the underlying assumption of a complete social network. These models assume that all of the adopters, $\bar{N}(t)$, (or informed individuals in case of the Dodson and Muller model) interact with all of the nonadopters $(\bar{N}(t)-N(t))$, or (uninformed individuals). This assumption is unrealistic since not all of the adopters or informed individuals are communicators of the product. Furthermore, these models have implicitly combined the effect of two distinct transfer mechanisms—individual experience and word-of-mouth—in the sense they assume that individual experience with the product is always communicated positively through word-of-mouth. It is very likely that the communicators of the product experience may transfer favorable, unfavorable, or indifferent messages through word-of-mouth. In other words, the fifth transfer mechanism—individual experience with the product—may not only influence the adopter's decision to repurchase, but may also influence potential adopters positively or negatively. What is now required is a comprehensive diffusion model which considers the customer flow across the untapped, potential, and current-market (consisting of triers or first-purchasers and second, third . . . nth repeat purchasers) identifying the communicators of the product and distinguishing the type of message conveyed. A start in this direction has been made in the innovative behavior model proposed by Midgley (1977).

Finally, diffusion models should possess parameters which are not abstract, but show the exact relationship between the product growth and marketing decision variable so that the effect of different marketing strategies on the product growth can be assessed. Furthermore, most of the current diffusion models are deterministic in that they provide point estimates. Hence, further extensions of these models should examine stochastic model formulations to provide a probabilistic range of sales estimates.

Empirical Validation and Evaluation. Regardless of the fact that some of the models have been available for some time, there has been relatively little material on their validity and reliability. There is a distinct "need" to know when the models "work" and when they do not. Furthermore, in order to establish the superiority of these models they need to be compared with other forecasting techniques to establish their (1) short- and long-run forecasting accuracy, (2) ability to identify future turning points, (3) time and cost requirements for model development and implementation, and (4) diagnostic power (Chambers, Mullick, and Smith 1971; Larreche and Montgomery 1977; Makridakis and Wheelright 1977; Wind 1974).

Conclusions

The objective of this paper has been to review and assess the state-of-the-art of diffusion models of new product acceptance. After reviewing the first-purchase models, a number of issues related to their further development and validation were presented. There have been some significant contributions to the development of these models but further development and continuous validation are needed to establish them as the useful tools of new product planning.

Appendix A

Customer Flow and Rate of Customer Flow Equations

The customer flow equations as presented in the text, are

$$S_1(t)+S_2(t)+N(t)=M(t) \qquad \text{(A1)}$$

where

$$S_1(t)=M(t)-\bar{N}(t) \qquad \text{(A2)}$$

$$S_2(t)=\bar{N}(t)-N(t) \qquad \text{(A3)}$$

The rate of the flow of customers across the three segments can be obtained by differentiating the above three equations, i.e.,

[7]Other repeat purchase models include those suggested by Fourt and Woodlock (1960), Eskin (1973), and Parfitt and Collins (1968). As compared to Fourt and Woodlock and Eskin, the Parfitt and Collins model is designed for long-run share predictions for a new brand introduced into an established market, rather than for a new product innovation. This model has been further refined by Shoemaker and Staelin (1976). Although none of the three models really use any of the innovation diffusion concepts, they do suggest the type of diffusion curves which may be employed to study the repeat purchases.

[8]Dodson and Muller illustrated that the advertising models proposed by Glaister (1974), Gould (1970), Nerlove and Arrow (1962), and Nicosia (1966) can be shown to be special cases or related to their model.

$$\frac{dS_1(t)}{dt} = \frac{dM(t)}{dt} - \frac{d\bar{N}(t)}{dt} \tag{A4}$$

$$\frac{dS_2(t)}{dt} = \frac{d\bar{N}(t)}{dt} - \frac{dN(t)}{dt} \tag{A5}$$

and
$$\frac{dN(t)}{dt} = \frac{dM(t)}{dt} - \frac{dS_1(t)}{dt} - \frac{dS_2(t)}{dt} \tag{A6}$$

In discrete time representation, rate equations are

$$s_1(t) = m(t) - \bar{n}(t) \tag{A7}$$

$$s_2(t) = \bar{n}(t) - n(t) \tag{A8}$$

$$n(t) = m(t) - s_1(t) - s_2(t) \tag{A9}$$

where

$s_1(t) = S_1(t+1) - S_1(t)$; $s_2(t) = S_2(t+1) - S_2(t)$;

$m(t) = M(t+1) - M(t)$; $\bar{n}(t) = \bar{N}(t+1) - \bar{N}(t)$; and

$n(t) = N(t+1) - N(t)$.

The customer flow equations, (A1)-(A3), and the rate equations, (A4)-(A6) or (A7)-(A9), form the basis for all the proposed diffusion models.

The basic diffusion models, (Bass 1969; Fourt and Woodlock 1960; Mansfield 1961; and the Gompertz curve) assume a constant total population of the potential customers, i.e., $\bar{N}(t)$ is constant or $\frac{d\bar{N}(t)}{dt} = 0$. Recalling equations (A1)-(A9), these models consider the following customer and rate equations:

Customer Flow:

$$S_2(t) = \bar{N} - N(t) \tag{A10}$$

Rate of Flow:

$$\frac{dN(t)}{dt} = -\frac{dS_2(t)}{dt} \tag{A11}$$

$$\text{or } n(t) = -s_2(t) \tag{A12}$$

Note in equations (A11) and (A12), that the decrease in the number of potential customers at any time t is equal to the simultaneous increase in the number of adopters at time t. Because of this relationship, the *basic* models are *primarily concerned with modeling $n(t)$ or $N(t)$*. Equations (A10)-(A12) also represent the customer flow and rate of customer flow for the extensions to the basic models proposed by Bass (1978), Horsky and Simon (1978), Lilien and Rao (1978), Peterson and Mahajan (1978), and Robinson and Lakhani (1975).

Modeling the product growth by considering the customer flow across all three segments of the diffusion process, the extensions proposed by Chow (1967), Dodson and Muller (1978), Lackman (1978), and Mahajan and Peterson (1978) implicitly assume that $M(t)$—the number of individuals in the total market—is constant. Recalling equations (A1)-(A9), these models consider the following customer flow and rate equations:

Customer Flow:

$$S_1(t) = M - \bar{N}(t) \tag{A13}$$

$$S_2(t) = \bar{N}(t) - N(t) \tag{A14}$$

$$N(t) = M - S_1(t) - S_2(t) \tag{A15}$$

Rate of Flow:

$$-\frac{dS_1(t)}{dt} = \frac{d\bar{N}(t)}{dt} \tag{A16}$$

$$\frac{dS_2(t)}{dt} = \frac{d\bar{N}(t)}{dt} - \frac{dN(t)}{dt} \tag{A17}$$

$$\frac{dN(t)}{dt} = -\frac{dS_1(t)}{dt} - \frac{dS_2(t)}{dt}. \tag{A18}$$

Equation (A17) implies that the increase in the number of potential customers at any time t is a net result of the decrease in the untapped market (individuals who have transferred from the untapped market to the potential market) and increase in the number of adopters (individuals who have transferred from the potential market to the current market). Because of this relationship, these *models are concerned with modeling $n(t)$ and $\bar{n}(t)$*. If $n(t)$ and $\bar{n}(t)$ are known, the flow and the rate of flow of customers across the three segments is completely specified.

In extending the basic diffusion models to include the third segment, untapped market, Mahajan et al. (1979) relax the assumption that $M(t)$—the number of individuals in the market—is constant. Equations (A1)-(A9) specify the customer flow and rate of flow for their model. In addition to modeling $n(t)$, *$\bar{n}(t)$, this model is also concerned with modeling $m(t)$*. The knowledge of $n(t)$, $\bar{n}(t)$, and $m(t)$ completely specifies the customer flow across the three segments.

Appendix B

Incorporating the effect of advertising into the growth model and considering the customer flow across all the three segments of the diffusion process, Dodson and Muller (1978) proposed the following rate equations for durables:

$$-\frac{dS_1(t)}{dt} = k_1 S_1(t)(S_2(t) + N(t)) + k_2 S_1(t) \tag{B1}$$

$$\frac{dS_2(t)}{dt} = k_1 S_1(t)(S_2(t) + N(t)) + k_2 S_1(t) - aS_2(t) \tag{B2}$$

$$\frac{dN(t)}{dt} = aS_2(t) \tag{B3}$$

where equation (B1) represents the decrease in the number of individuals in the untapped market (i.e., individuals who have transferred from the untapped market segment to the potential market), equation (B2) represents the net increase in the number of potential customers (number of individuals who have transferred from the untapped market to the potential market minus the number of individuals who have transferred from the potential market to the current market) and equation (B3) represents the increase in the number of adopters. Note that in their model

$$\frac{dS_1(t)}{dt}+\frac{dS_2(t)}{dt}+\frac{dN(t)}{dt}=0. \tag{B4}$$

That is, recalling the general rate equation (A6) for the flow of customers across the three market segments, it is clear that Dodson and Muller assume $\frac{dM(t)}{dt}=0$, i.e., the total number of individuals in the market, $M(t)$, is constant. Second, because of this relationship the customer flow across the three segments is completely specified by *two* rate equations. That is, if $\frac{dS_1(t)}{dt}$ and $\frac{dN(t)}{dt}$ are known,

$$\frac{dS_2(t)}{dt} = -\frac{dS_1(t)}{dt} - \frac{dN(t)}{dt}. \tag{B5}$$

Now, from equations (A13), (A14), and (A16), since $S_1(t) = M - \bar{N}(t)$, $S_2(t) = \bar{N}(t) - N(t)$, and $-\frac{dS_1(t)}{dt} = \frac{d\bar{N}(t)}{dt}$, the model proposed by Dodson and Muller, equations (B1) and (B3), reduces to the following rate equations:

$$\frac{d\bar{N}(t)}{dt} = k_1\bar{N}(t)(M-\bar{N}(t))+k_2(M-\bar{N}(t))$$

$$\frac{dN(t)}{dt} = a(\bar{N}(t)-N(t)) \tag{B6}$$

The discrete formulation of equation (B6) is given by equation (26).

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