ADVERTISING PULSING POLICIES FOR GENERATING AWARENESS FOR NEW PRODUCTS

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The question of whether a pulsed advertising policy is superior to an even policy (constant spending over time) is of practical relevance to both advertising practitioners and model builders. This paper presents an analytical model that can be used to analyze the impact of the various pulsing and even policies on awareness. In addition to establishing the relationships between the various advertising policies analytically, an application of the proposed model to the actual Zielske's data is included.
(Awareness Forecasting; Advertising Policies; Pulsing)

1. Introduction

In launching a new product, advertisers must make a choice between a campaign based on continuity versus pulsing. Continuity is achieved by scheduling exposures evenly within a given time period. Pulsing refers to scheduling exposures unevenly over the same time period. Thus 52 exposures could be scheduled continuously at one a week throughout the year or pulsed in bursts of 13 exposures in each of four months. For media planners the key decision is, given a certain advertising budget or the number of exposures that can be bought in a medium for a certain time period T, how should the successive exposures be scheduled?

The question of scheduling successive exposures has been of interest to both advertising practitioners and model builders (Little 1979, Aaker and Myers 1982). In order to analyze this question, most of the empirical studies, for example, have been concerned with the rate at which consumers can be made to remember or learn advertising and the rate at which they forget it. That is, how much of awareness (or whatever is of interest) is retained from period to period without further exposure? One of the first psychologists to study learning and forgetting empirically was Ebbinghus (1913). In a series of experiments, he related retention to repetition and to the time since the learning experience. Despite their limitations, his experiments, for example, suggest that: (1) exposures should be spread out over time and not bunched together, (2) there are diminishing returns to repetition and (3) the amount of relearning increases with the amount of time elapsed since the original learning occurred, however, this increase grows smaller over time (exponential forgetting).

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Since the publication of the results by Ebbinghus, a number of other empirical studies have also tried to replicate his results (Strong 1914, Zielske 1959, Strong 1974, Ackoff and Emshoff 1975, Rao 1970). More recently, reanalyzing Zielske's data Simon (1979, p. 419) found that "a given advertising budget is most efficient if it is spread out over the maximum period rather than concentrated in a single burst. Several bursts will be better than one burst but worse than an even spread, by implication".

It is interesting to note, however, that although the empirical studies support the notion of spreading out the exposures over time, they do not provide any normative guidelines as to the number and/or the timing of the exposures in a given time period. A practical suggestion, however, has been made by Krugman (1972) who advocates that 3 exposures may be enough to stimulate action.

Other than the empirical studies, an interesting analytical perspective on the scheduling of successive exposures has been provided by Sasieni (1971). In order to understand his results, consider Figure 1 which represents an S-shaped response curve relating the level of advertising spending, u, and advertising effectiveness (say, level of awareness), $f(u)$ (Johansson 1970). The S-shaped curve captures both the phenomena of increasing and decreasing marginal returns to the various levels of advertising spending. The point u^*, point of inflection, splits the curve into two parts, the bottom part being convex (representing increasing marginal returns) and the upper part being concave (representing decreasing marginal returns). Sasieni (1971) and Lodish (1971a) argue that a firm will not advertise at the convex part of the effectiveness function since if it advertises at $\alpha\bar{u}$ for $0 < \alpha < 1$ and $\bar{u}$ greater than u^* (see Figure 1), the effectiveness of such an advertising policy will be $f(\alpha\bar{u})$. This effectiveness value, $f(\alpha\bar{u})$, is smaller than $\alpha f(\bar{u})$ that the firm
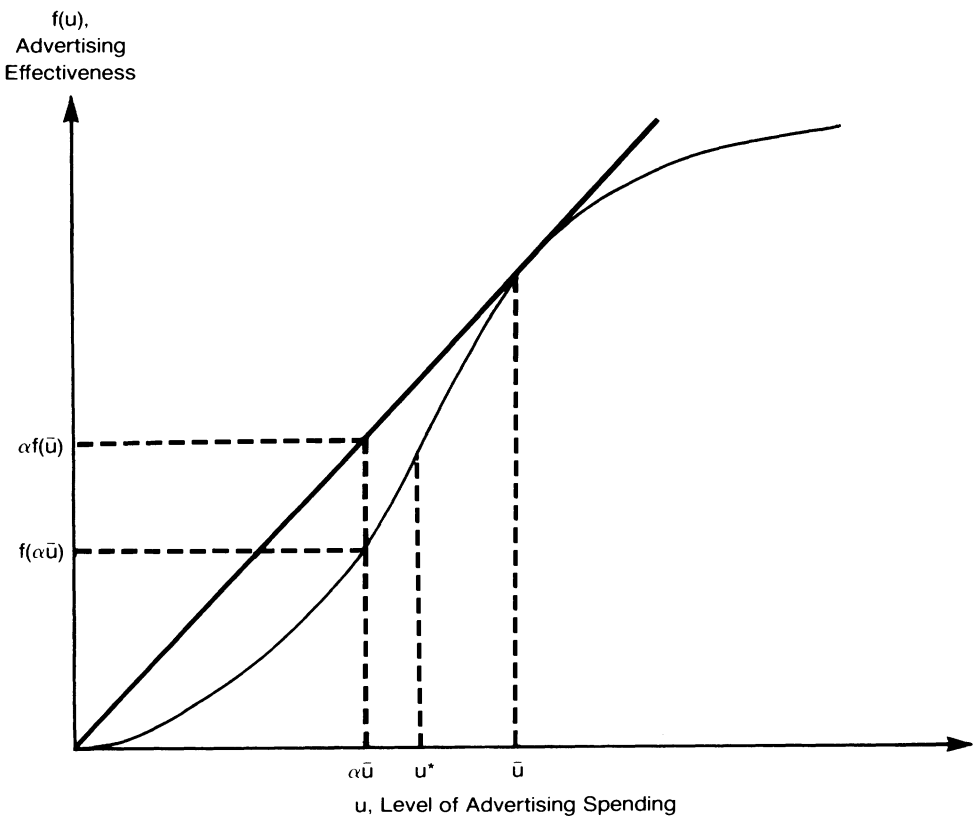

FIGURE 1.   The S-Shaped Advertising Response Function.

can achieve by *chattering* between $u = \bar{u}$ and u = zero. That is, to achieve the maximum impact, the firm must alternate between the two points ($u = \bar{u}$ and $u = 0$) without actually spending any positive amount of time at any of two points. Since the firm can achieve the linear response (by connecting $u = 0$ and $u = \bar{u}$), marginal returns, therefore, do not increase anywhere.

Although these results are analytically elegant, in practice, however, the firm cannot chatter. The chattering policy actually implies having an infinite number of bursts in a given time period. Now, given the empirical results, suggesting that several bursts are better than one, and the analytical results by Sasieni, the chattering policy, the practical question is: for a given product in a given target market, and a given advertising budget for a given time period, how should the successive exposures be scheduled to achieve the maximum possible impact of advertising. That is, is a pulsed advertising schedule superior to an even schedule (constant spending over time)? If yes, what should be the number and timing of these pulses?

In order to investigate this issue, this paper presents an analytical model that can be used to analyze the impact of the various advertising pulsing and even policies. More specifically, given the importance of generating awareness to influence sales for new products, the focus of this paper is restricted to the examination of the impact of advertising policies on awareness.

The format of this paper is as follows. The next section provides the analytical model. §3 includes an analytical comparison of the various advertising policies. §4 contains an application of the proposed model to the actual Zielske's data provided by Simon (1979) and also includes results of a comprehensive simulation study to develop certain general guidelines to schedule successive exposures. §5 concludes with model limitations, and some further comments on the relative contribution of the proposed model to the literature.

2. Problem Formulation

It was indicated earlier that a firm that wishes to advertise to create awareness has to make a choice between a pulsed or an even advertising schedule. In the literature, however, different policies have been included under "pulsing". Before presenting the analytical formulation, five specific policies (including the even policy) examined in this paper are defined below:[1]

(a) *Blitz* or one pulse policy at which the firm concentrates *all* its efforts in some initial periods.

(b) *Pulsing* policy in which the firm alternates between high and zero levels of advertising.

(c) *Chattering* policy which is a theoretical policy in which the firm alternates an infinite number of times during the finite planning horizon between high and zero levels of advertising.

(d) *Even* policy at which the firm advertises at some constant level throughout the planning horizon.

(e) *Pulsing/Maintenance* policy in which the firm combines any of the above policies with a low level of advertising, usually a maintenance level. For example the firm can alternate between high and low levels of advertising.[2]

These policies (except for chattering) are depicted in Figure 2.

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[1] Note that our terminology differs from Simon (1979) and Zielske (1959). They refer to blitz as pulse and pulsing as spacing.

[2] As will be indicated in §3, the reason that we include all pulsing/maintenance policies into one category is that our results indicate that, in the presence of an S-shaped advertising response function, pulsing/maintenance policies are inferior to pulsing policies.
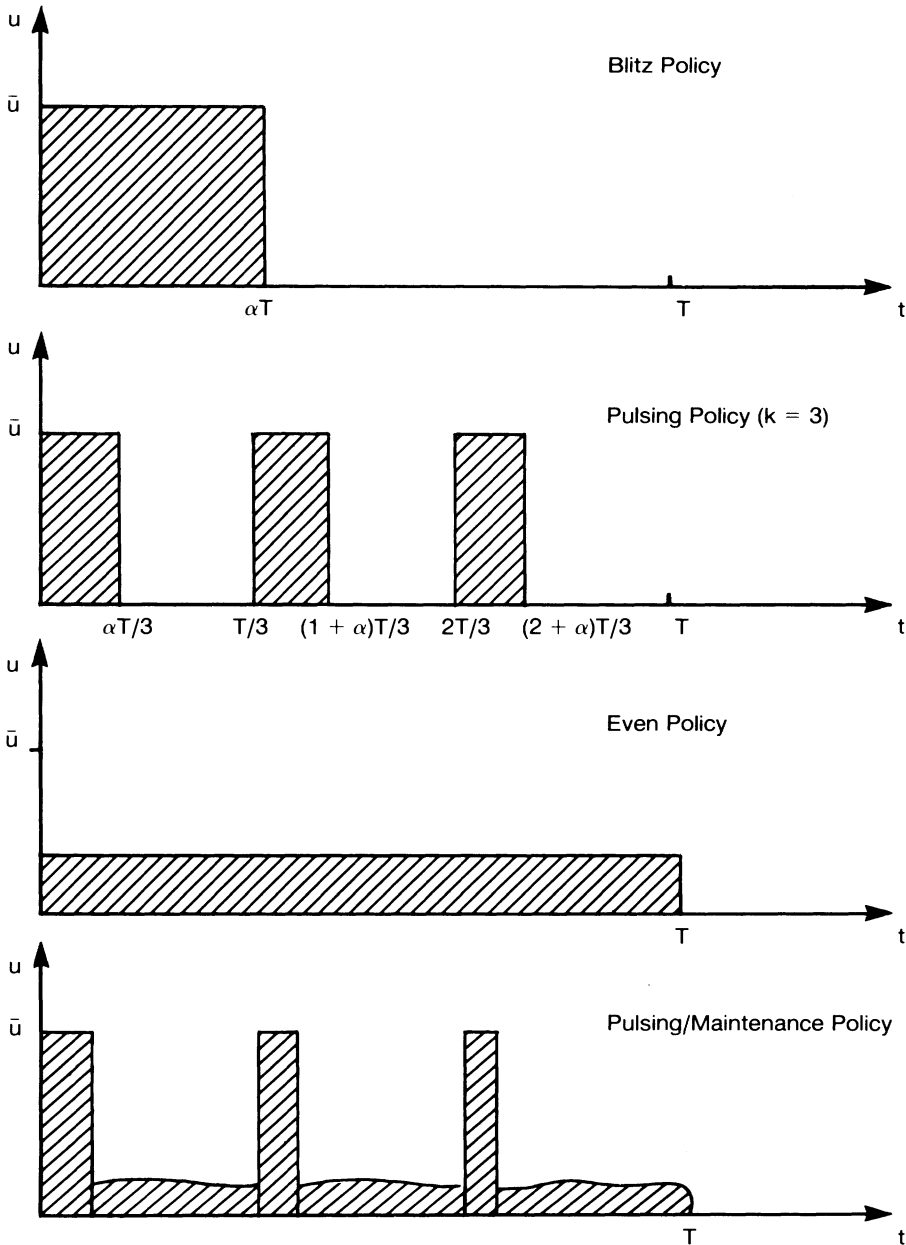
FIGURE 2.   Advertising Policies.

Clearly, blitz and chattering policies can be thought of as special cases of pulsing policy when the number of pulses is one and infinite, respectively. We have chosen to specify these as separate policies, mainly because of the special attention these policies have received in the literature.

2.1. *Pulsing Policy*

In order to formally define a pulsing policy with a given budget B for a time period T, let (see Figure 2)

$\bar{u}$ = a given level of advertising spending (to be specified shortly).

$\alpha = B/\bar{u}T$ = proportion of time (out of time period T) in which the firm advertises at level $\bar{u}$.

k = number of times (an integer value) the firm switches from an advertising level of $\bar{u}$ to zero; i.e., number of pulses.

Given the above terminology, a pulsing policy can be described now. For example, for $k = 1$,

$$u = \begin{cases} \bar{u} & \text{for} & 0 \leq t < \alpha T, \\ 0 & \text{for} & \alpha T \leq t \leq T. \end{cases}$$

For $k = 2$

$$u = \begin{cases} \bar{u} & \text{for} & 0 \leq t < \alpha T/2 & \text{and} & T/2 \leq t < (1 + \alpha)T/2, \\ 0 & \text{for} & \alpha T/2 \leq t < T/2 & \text{and} & (1 + \alpha)T/2 \leq t \leq T. \end{cases}$$

In general, then, a k-pulsing policy can be defined as:

$$u = \begin{cases} \bar{u} & \text{for} & iT/K \leq t < (i + \alpha)T/k, & 0 \leq i \leq (k - 1), \\ 0 & \text{for} & (i + \alpha)T/k \leq t < (i + 1)T/k & \text{and for} & t = T, & 0 \leq i \leq (k - 1). \end{cases} \tag{1}$$

According to the above formulation, irrespective of k, the firm advertises at $\bar{u}$ for αT periods and at zero level for the rest of the periods, i.e., for $(1 - \alpha)T$ periods. Note that the width of each pulse i.e., the length of time the firm advertises in each cycle is given by $W = \alpha T/k$. Thus the larger α and T are, the larger is the width of each pulse. However, the more pulses the firm employs, the smaller does the width become. Its total budget is given by:

$$\int_0^T u(t)dt = \bar{u} \sum_{i=0}^{k-1} ((i + \alpha)T/k - iT/k) = \bar{u}\alpha T = B.$$

Thus regardless of what k is, the firm depletes its budget B by (or before) the end of the planning horizon T.

2.2. Linking Pulsing to Awareness

For a new product, awareness in a target market can be generated by advertising, free samples, word-of-mouth communication and other marketing activities such as in-store displays. However, restricting our scope only to advertising, in order to examine the impact of the various advertising policies, it is first necessary to specify an awareness forecasting model and to develop an overall effectiveness measure of an advertising campaign.

In an empirical comparison of six awareness forecasting models embedded in the various new product introduction models, Mahajan, Muller and Sharma (1984) have documented that the learning and the forgetting phenomena can be captured by the following simple model (dropping time subscript t for convenience):

$$dA/dt = f(u)(1 - A) - bA \tag{2}$$

where A is the fraction of the target market aware of the new product at time t, f is the advertising response function, and b is the decay or forgetting parameter. Note that in equation (2), the term $f(u)(1 - A)$ captures learning and the term bA captures forgetting. In order to solve equation (2), it is necessary to specify an initial value condition and $f(u)$. For the initial value condition, for simplicity, we assume that $A(t = 0) = 0$. In addition, we specify the response function $f(u)$ to be an S-shaped function such as the one given in Figure 1.[3] Note in Figure 1 that $\bar{u}$ is obtained by the tangency point of the

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[3] It should be noted that a convex advertising cost function used by many researchers is equivalent to a concave advertising response function.

ray from the origin to $f(u)$. Although formally defined in the next section, it is important to point out here that $\bar{u}$ represents the threshold level of advertising. The reason that we specify $f(u)$ to be an S-shaped function is that the analytical results presented in the next section indicate that pulsing is optimal *only* under an S-shaped advertising response function and indeed pulsing is *not* optimal under a concave response function. More will be said about this assumption in the next and the last sections of the paper.

Now, linkage of the pulsing policy equation (1) with the awareness forecasting equation (2) suggests that during the periods in which the firm advertises, the awareness evolves according to equation (2) with $f(u) = f(\bar{u})$. Letting $f(\bar{u}) = x$, substitution of x into equation (2) yields the following solution of equation (2) for the periods in which the firm advertises at $\bar{u}$ level of spending:

$$A(t) = A(iT/k)e^{(x+b)(iT/k-t)} + x(1 - e^{(x+b)(iT/k-t)})/(x + b). \tag{3}$$

Similarly, during the periods in which the firm does not advertise, $f(u) = 0$ and the solution of equation (2) is:

$$A(t) = A((i + \alpha)T/k)e^{b((i+\alpha)T/k-t)}. \tag{4}$$

Notice that equations (3) and (4) essentially capture the evolution of awareness under the various pulsing schedules. In fact, for an empirical application, given α, T and k, equations (3) and (4) can be used simultaneously to estimate b and x by using the standard nonlinear estimation procedures. Once b and x have been estimated, the impact of the various pulsing policies on the evolution of awareness can be easily estimated by varying the value of k in equations (3) and (4). In fact, this is the procedure which is used to examine the Zielske's data in §4.

2.3. *Measuring Total Impact of An Advertising Policy*

The objective of the firm is to generate awareness via equation (2) by utilizing its budget most effectively. However, different pulsing policies can generate different distributions of awareness levels over time. As pointed out by Simon (1979), one possible approach, and also employed in this paper, to measure the overall impact of an advertising policy is to measure the area under the distribution of awareness levels over time created by the specific advertising policy. This measure, to be denoted by R_k, in fact, is simply the total awareness generated during the time period T. However, as noted by Simon (1979) in Zielske's data, since different pulsing policies can result in different final level of awareness at time T, this level has to be taken into account when comparing the various pulsing policies. The method to do it is to compute additional total awareness after time T until the awareness level at time T vanishes. Although the firm can clearly advertise after T (say at a maintenance level), for comparison purposes only, we assume that the firm does not advertise after time T.[4] That is, the awareness level at time T decays indefinitely according to equation (2) with $f(u) = 0$.[5] That is,

$$dA/dt = -bA. \tag{5}$$

[4] This, of course, is a restricted assumption. As discussed by Little and Lodish (1969), depending on assumptions of what happens after the time horizon is over, the relative value of different policies may change since the value of end conditions will change. For further discussion on end effects, see Little and Lodish (1969).

[5] We thus assume symmetry between the forgetting rates for the periods during which the firm advertises and the periods during which it does not advertise. Since $f(0) = 0$, this assumption makes equations (2) and (5) internally consistent. The pulsing model reported by Mesak (1985) assumes two different forgetting parameters (for a simple Nerlove-Arrow model) and suffers from internal inconsistency. It should be noted that this is not the *asymmetry* reported by Little (1979) between sales growth and decay. The asymmetry reported by him can very well be explained in our model since the decay depends upon the decay parameter b, while growth, in addition, also depends upon the function $f(u)$ and thus on advertising intensity u.

The total awareness generated by a k-pulsing policy, therefore, is given by:

$$R_k = \int_0^T A(t)dt + \int_T^\infty A(t)dt \qquad (6)$$

where $A(t)$ follows equation (2) (or equations (3) and (4)) for $0 \le t \le T$ and equation (5) thereafter.

Solving equation (5), substituting into equation (6) and performing the integration yields:

$$R_k = \int_0^T A(t)dt + A(T)/b. \qquad (7)$$

The second term in equation (7) summarizes the value of total awareness at time T.[6] For a k-pulsing policy, the first term in equation (7) can be evaluated by using equations (3) and (4). In fact, the following results are derived in Appendix A for the various advertising policies ($x = f(\bar{u})$):

(a) *Blitz*, $k = 1$:

$$R_1 = \alpha x T/(x + b) + x^2(1 - e^{-\alpha(x+b)T})/b(x + b)^2. \qquad (8)$$

(b) *Pulsing*, for any k:

$$R_k = \alpha x T/(x + b) + BLx^2/b(x + b)^2 \qquad \text{where} \qquad (9)$$

$$B = (1 - e^{-\alpha(x+b)T/k})/(1 - e^{-(\alpha x+b)T/k}) \qquad \text{and}$$

$$L = B(1 - e^{-(\alpha x+b)T})e^{-(1-\alpha)bT/k} + k(1 - e^{-(1-\alpha)bT/k}).$$

(c) *Chattering*, $k = \infty$:

$$R_\infty = \alpha x T/(\alpha x + b) + (\alpha x)^2(1 - e^{-(\alpha x+b)T})/b(\alpha x + b)^2. \qquad (10)$$

Equations (8), (9) and (10) provide the total awareness values for comparing blitz, pulsing and chattering policies, respectively. However, in order to compare these policies with the other two policies, even and pulsing/maintenance, the corresponding total awareness values for these two policies need to be developed. Fortunately, however, it is not necessary to derive the total awareness value for pulsing/maintenance. It is shown analytically in the next section that for an S-shaped advertising response function, the pulsing/maintenance policy is always inferior to the corresponding pulsing policy. That is, the total awareness value for the pulsing/maintenance value is always smaller than the total awareness value for the corresponding pulsing policy.

In order to derive the total awareness value for the even policy, assume that the firm advertises at a constant level of advertising spending $\alpha\bar{u}$. Its total expenditures are $\alpha\bar{u}T$ and thus it is a comparable policy since the firm depletes its budget by the end of the period T. Let the advertising function at this level be equal to y, i.e., $y = f(\alpha\bar{u})$. Let total awareness be denoted by R_c. Substitution of the solution of equation (2) for $y = f(u)$ into equation (7) and further integration yields the following total awareness value for the even policy (where $y = f(\alpha\bar{u})$):

(d) *Even*:

$$R_c = yT/(y + b) + y^2(1 - e^{-(y+b)T})/b(y + b)^2. \qquad (11)$$

For an empirical application, given α and T, and estimates of b and x from equations (3) and (4), the impact of the various advertising policies on awareness can be assessed by using equations (8)–(11).

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[6] The total awareness can be easily transformed into average awareness by dividing it by the number of time periods T. If, however, one wishes to take into account the residual awareness at time T, as we have done in equation (7), the result is an adjusted average awareness, i.e., R_k/T.

3. An Analytical Comparison of Advertising Policies

Having formally defined the even and pulsing policies and a measure of their impact on awareness, this section analytically examines the relationships between the five specific policies described in the last section: blitz, pulsing, chattering, even and pulsing/maintenance. The relationships among these policies will be stated in terms of five propositions. All of the propositions assume that the advertising response function is either S-shaped or concave. In addition to showing the relationships among the policies, the basic theme of this section is to establish that pulsing is optimal *only* under an S-shaped advertising response function and to provide a formal definition of $\bar{u}$ as the threshold level of advertising. Most of the analytical relationships established in this section will be demonstrated empirically for Zielske's data in the next section.

A formal definition of domination, adopted from game theory, is needed for stating our results. That is, strategy a *dominates* strategy b, if when the firm employs strategy a, its objective function (in our case total awareness) is larger than under strategy b. A set of strategies A *dominates* a set of strategies B if for any strategy in B, there exists a dominating strategy in A.

The following propositions summarize the relationships among the five strategies described earlier.

PROPOSITION 1. *Under an S-shaped advertising response function, a blitz policy is dominated by a chattering policy.*

PROOF. See Appendix B for a proof.

PROPOSITION 2. *Under an S-shaped advertising response function, pulsing policy dominates blitz but is dominated by chattering.*

We have not been able to show analytically that R_k increases in k, i.e., that total awareness increases as the firm increases the number of pulses of its advertising. Such a proposition would, of course, replace Proposition 1 and relate both policies to pulsing policy. Existence of such a proposition, however, was examined by simulating the sensitivity of R_k, equation (9), for the various values of the relevant parameters. The simulation results, reported in the next section, indicate that for *all values* of the parameters that we have tried, R_k indeed increases in k. Thus we can conclude (with no formal proof) that *pulsing* policy dominates blitz but is dominated by *chattering*.

PROPOSITION 3. *An even policy is dominated by a chattering policy if and only if the advertising response function is S-shaped.*

PROOF. First observe that $f(u)$ is S-shaped if and only if $y = f(\alpha\bar{u}) < \alpha f(\bar{u}) = \alpha x$. Note also from equations (10) and (11) that when $y = \alpha x$, $R_c = R_\infty$ and that R_c is increasing in y.   Q.E.D.

Note the "only if" part of the proposition. It states that in case that the function is concave, an even strategy is better than chattering and therefore dominates blitz, pulsing and pulsing/maintenance. The relationship between the even and the blitz strategies is more complex as is shown in the next proposition.

PROPOSITION 4. *Under an S-shaped advertising response function, an even policy is dominated by a blitz policy for small values of $y = f(\alpha\bar{u})$ and dominates a blitz policy for large values of $y = f(\alpha\bar{u})$.*

PROOF. The largest value y can take is αx, and then $R_c = R_\infty > R_1$ by Propositions 2 and 1. For small values of y, construct a series of functions $f(u)$ such that for a fixed value of α and x, $y = f(\alpha\bar{u})$ approaches zero. This is done by increasing the curvature of $f(u)$. Observe then that R_1 remains fixed but R_c approaches zero.   Q.E.D.

Thus the exact shape of the S-shaped advertising response function $f(u)$ determines which of the two strategies is superior. For a function which is similar to a linear function in its convex part, an even policy is better. For a function which is a step-like function, a blitz is superior.

Observe that pulsing/maintenance policy is in effect a whole collection of strategies such as a blitz followed by a maintenance level or pulsing between high and low levels of advertising or even a chattering policy between high and low levels.

PROPOSITION 5. *The pulsing/maintenance policy set is dominated by the blitz, pulsing and chattering policies if and only if the advertising response function is S-shaped.*

PROOF. For every pulsing/maintenance policy observe the first maintenance level. By Proposition 3 it is dominated by a chattering policy for this duration. By Proposition 2, there is a pulsing policy with k pulses that dominates it as well.   Q.E.D.

Observe that if the function $f(u)$ is concave, the proposition states that pulsing or chattering is not optimal. Indeed the optimal policy in this case is the standard one of a blitz followed by a maintenance policy.

With the help of the above propositions it is now possible to formally define a threshold level in an S-shaped function.

We need first a definition of efficiency of a level of advertising. In any advertising policy, a level u_1 is *inefficient* if for all budget levels, there exists another level u_2 that dominates it.

A level of advertising will be denoted as a *threshold* level if it is the lowest efficient level of advertising.

In Figure 1, $\bar{u}$ is the threshold level that is obtained by the tangency point of the ray from the origin to the response function $f(u)$. Clearly it is already at a concave part of $f(u)$ i.e., where there are decreasing marginal returns. Since it is the tangency point, it can be found by solving the following equation:

$$\bar{u}f'(\bar{u}) = f(\bar{u}). \tag{12}$$

Equation (12) formally defines $\bar{u}$ in equation (1), where $f'(\bar{u})$ is the first derivative of $f(\bar{u})$.

4. An Application

In order to illustrate the application of the proposed model, equations (3) and (4), Zielske's data reported by Simon (1979) are reanalyzed. In 1959, Zielske (1959) tried to measure consumer recall and forgetting rate of advertising by mailing 13 different reprints of newspaper advertisements for an ingredient food to two groups of randomly selected women. One group received an advertisement weekly for 13 weeks and then received no more ads for the rest of a year. The second group received the same 13 mailings but at intervals of four weeks during the year. Throughout the year, advertising recall was measured in both groups by telephone interviews (for further details and shortcomings of these data, see Simon 1979). Using the total awareness as the measure of advertising impact, i.e., R_k in equation (9), and determining this value graphically Simon concluded that pulsing (every four-weeks schedule) is better than blitz (weekly schedule). From these results he also *inferred* that even policy is better than pulsing.

In order to examine his conclusions, we limit our analyses to only the first group.[7] For

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[7] Unless smoothing is done to the data for the second set of 14 groups, which, for example, has been done by Zielske (1959) and Simon (1979) and used by Lodish (1971b), it is not possible to use Zielske's second data set. The reason for this is that Zielske sent same number of ads at four-week intervals to two groups at a time resulting in seven independent values for the number of ads sent. However, out of the two groups which received the same number of ads, awareness was measured at the end of the first week since the mailing of the last ad
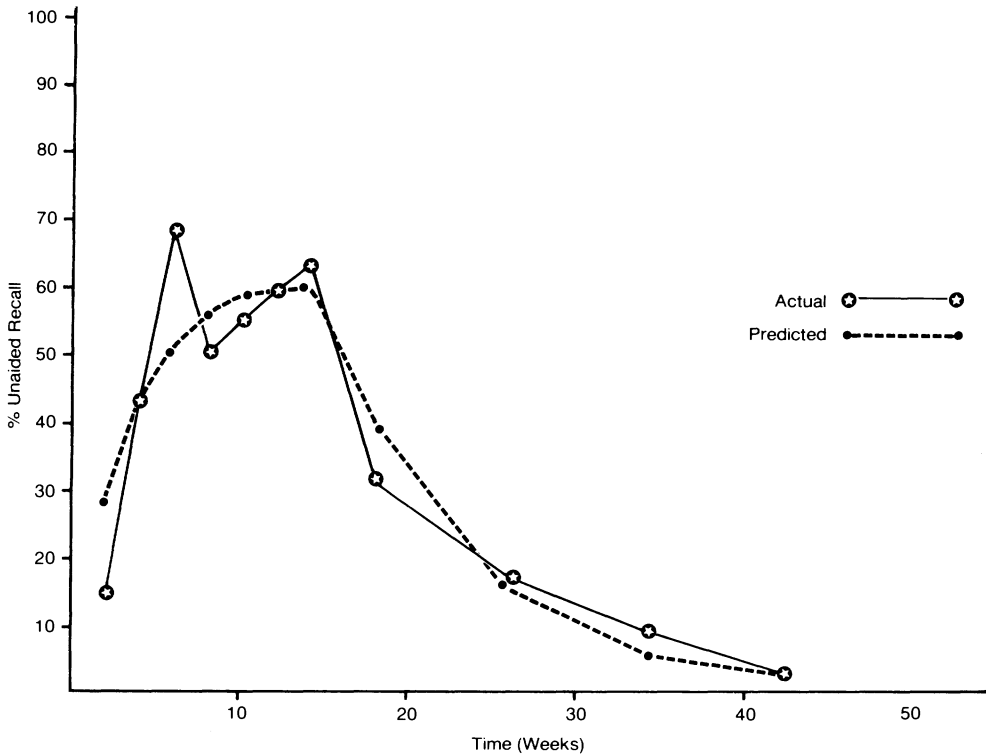
FIGURE 3. Actual and Predicted Values for the Zielske's Data for Group 1.

this experiment since one blitz of 13 exposures was used for the entire 52 weeks, $k = 1$, $\alpha = \frac{13}{52} = \frac{1}{4}$ and $T = 52$. A nonlinear estimation algorithm was used to estimate the parameters b and x in equations (3) and (4) yielding $b = 0.12201$, $x = 0.18216$ and explained variance $= 0.96$. Figure 3 plots the actual and predicted values of awareness. Given these encouraging results, it can be concluded that the model describes the learning/forgetting process very well. In order to assess the impact of the various advertising policies, equations (8), (10) and (11) are used to estimate the total awareness providing the following results:

    Blitz: $R_1 = 10.6643$.

    Chattering: $R_\infty = 14.7301$.

    Even: $R_c = 13.339$ (for $y = 0.04$), $R_c = 7.4864$ (for $y = 0.02$).

For the pulsing policy, using equation (9), Figure 4 plots the values of R_k for the various values of k. Note from Figure 4 that total awareness monotonically increases with k with $R_{26} = 14.7110$. The following comments are warranted from these results:

    • As expected chattering policy yields the maximum value of total awareness (assuming an S-shaped response function).

    • Depending upon the value of y (which is less than $\alpha x = 0.045$), the even policy may or may not be better than the blitz policy. For example, for $y = 0.04$, the even policy is better than the blitz policy. However, for $y = 0.02$, the even policy is inferior to the blitz policy. Similarly, depending upon the number of pulses used in the pulsing policy, the

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for the first group and at the end of the fifth week since the mailing of the last ad for the second group. If it is assumed that across the groups awareness evolves over time collectively as a function of number of ads, then the second awareness observation at the end of the fifth week is contaminated for the 14 groups as a whole since an additional ad is sent at the end of the *fourth* week to the two groups receiving the next treatment value of the number of ads.
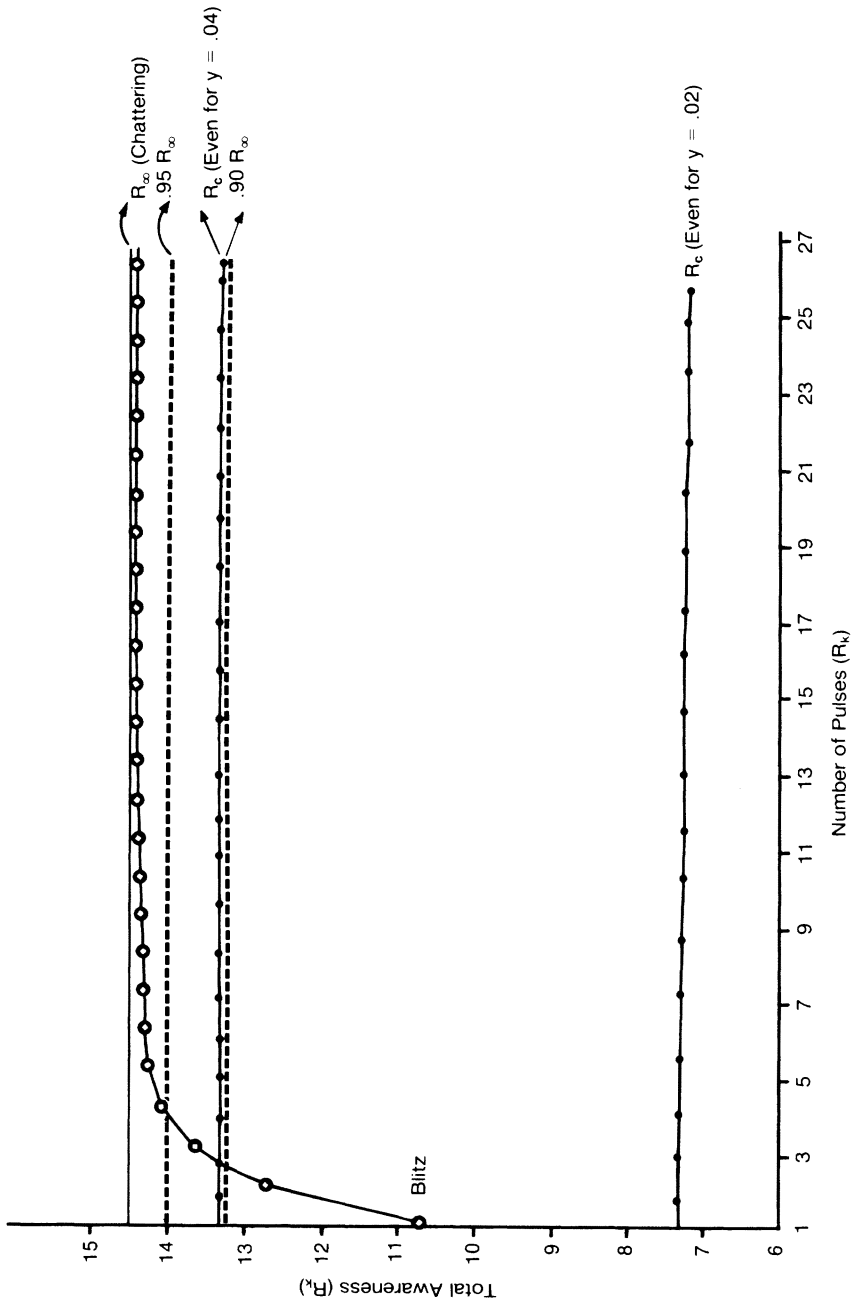
FIGURE 4. Total Awareness for the Various Advertising Policies for the Zielske's Data for Group 1.

even policy may or may not be better than the pulsing policy. For example, for $y = 0.04$, $R_c = 13.339$. This level of total awareness, however, can be achieved with $k > 2$ ($R_3 = 13.5733$). For $k = 2$, pulsing policy is inferior to the even policy ($R_2 = 12.6422$).

• The pulsing policy always yields better results than the blitz policy. But the pulsing policy is inferior to the chattering policy. However, 90% *of the efficiency of the chattering policy* ($R = 0.90R_\infty = 13.2571$) *can be achieved with k less than 3*. That is, three pulses can generate more than 90% of the impact that can be achieved by the chattering policy. Similarly, 95% of the impact of the chattering policy can be achieved with k less than four and 99% with k less than 10.

• Based on these results for the Zielske's first group, the average recall for the blitz policy ($k = 1$) is 20.51% (($R_1 \times 100$)/52 = (10.6643×100)/52). Similarly, for a pulsing policy of $k = 13$ (monthly spaced advertising over the year), $R_{13} = 14.6543$ and the average recall is 28.18% yielding an advantage of 37.39% over the blitz policy. It is interesting to note that these calculations support the results reported by Zielske and differ from the results reported by Simon. As reported by Zielske (p. 241), monthly spaced advertising to his second group ($k = 13$) produced 29% average recall over the year compared with 21% for the 13 weekly advertisements to the first group ($k = 1$), an advantage of 38.09%. Using "eye-and-hand" analyses, Simon, on the other hand, calculated an average recall of 40.96% for the monthly spaced advertising (he estimated $R_{13} = 21.30$) as compared to 20.69% for the 13 weekly advertisements (estimated $R_1 = 10.76$), an advantage of 97.97%. It is hard to speculate the overwhelming advantage of pulsing over blitz calculated by Simon. His calculations may have been affected by the smoothing performed on the data for the second group.

The above results from Zielske's first group suggest that the pulsing policy is always better than the blitz policy. The pulsing policy is, however, inferior to the chattering policy. The even policy may or may not be better than the blitz or the pulsing policies. Finally, from a practical point of view, given the data for the first group, less than three pulses can generate 90% of the impact generated by the chattering policy. It should be noted, however, that the implicit assumption (although supported by the earlier mentioned five propositions and the above empirical results) underlying our application to the Zielske data is that the response function is an S-shaped curve.

4.1. *A Simulation Study*

As depicted in Figure 4, Zielske's data clearly suggest that total awareness increases with the number of pulses. In addition, for $T = 52$, $\alpha = \frac{1}{4}$, $b = 0.12$ and $x = 0.18$ for his first group, the results suggest that less than three pulses can generate 90% of the impact generated by the chattering policy. In order to further examine these results and to support Proposition 2 in the last section, a simulation study was conducted. Assuming $T = 52$, the simulation study involved estimation of the value of total awareness, R_k in equation (9), for the various values k, α, b and x. Twelve different values were assumed for α, i.e., $\alpha = 0.01, 0.03, 0.05$, and 0.1 through 0.9 in increments of 0.1. Similarly, nine different values were selected for b and x, i.e., b, x, = 0.1 through 0.9 in increments of 0.1. For each combination of α, b and x (for a total number of 972 such combinations), total awareness R_k was estimated by varying k from one through 26. In addition, for each combination R_∞ was also estimated by using equation (10).

The estimated value of R_k for each combination of α, b and x clearly indicated that the total awareness increases with the number of pulses supporting Proposition 2 in the last section. Furthermore, for each combination, the minimum number of pulses required to achieve 90% of the impact of chattering policy was also estimated. Table 1 summarizes these results for the 972 such combinations. The following observations are warranted from this table:

TABLE 1

Simulation Results

Minimum Number of Pulses Required to Achieve 90% of the Impact of Chattering Policy	Number of Cases Generated by Simulation for the Various Values of α (the Proportion of Time that the Firm Advertises in the Planning Horizon T)												Total Number of Cases	
	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	No.	%
1	42	13	9	5	4	5	5	8	12	20	73	81	277	28.5
2	39	19	11	5	5	4	5	4	5	12	8		117	12.0
3		24	16	11	3	2	3	8	10	18			95	9.8
4		17	17	13	10	9	7	5	13	18			109	11.2
5		8	14	9	6	7	9	11	13	13			90	9.3
6			10	11	11	9	9	9	11				70	7.2
7			4	10	10	7	10	11	8				60	6.2
8				8	6	9	6	8	7				44	4.5
9				5	8	8	8	6	2				37	3.8
10				4	6	6	6	7					29	3.0
11					6	5	6	3					20	2.1
12					4	5	5	1					15	1.5
13					2	3	2						7	0.7
14						2							2	0.2
Total	81	81	81	81	81	81	81	81	81	81	81	81	972	100

(a) A maximum of 14 pulses is required to cover all the possible values of α, b and x included in the study.

(b) Approximately 90% of cases (out of 972) are covered by eight pulses.

(c) As expected (not shown in Table 1), for the same values of α and b, number of pulses increases as x increases; and for same values of α and x, number of pulses increases as b increases.

(d) For same values of b and x, number of pulses increases generally up to $\alpha = 0.2$ through 0.5 and decreases thereafter. (This trend is also reflected in Table 1.)

The simulation results confirm what has been the focus of a number of clinical studies. That is, the total awareness increases with the number of pulses. However, the results clearly suggest that the optimal number of pulses needed in a particular setting depends upon the values of α, the proportion of time (out of T) that a firm elects to advertise, b, the forgetting parameter, and x, the value of the advertising response function at the threshold level $\bar{u}$.

5. Discussion

A firm that wishes to advertise to create awareness has to make a decision regarding the successive scheduling of exposures. The key question that needs to be investigated is whether a pulsed advertising policy is superior to an even policy. If yes, what should be the timing and the number of the pulses. This paper has proposed a simple analytical model that can be used to assess the impact of the various advertising policies on awareness and to determine the "optimal" number of pulses for a particular data setting.

The underlying assumption in the proposed model is that the advertising response is an S-shaped function (see Figure 1). In fact, the analytical results presented in §3 indicate that pulsing is *optimal* only under an S-shaped response function. How general is this result and how sensitive it is to the particular form of the awareness model, equation (2), used in this paper deserve some further comments.

As was indicated earlier, although the awareness model, equation (2), does not include the effect of the other awareness generating stimuli such as word-of-mouth, free samples

or coupons, it is a general formulation that has been used in a number of new product introduction models to relate advertising to awareness (see Mahajan, Muller and Sharma 1984). It is also equivalent in its dynamics to the models proposed by Vidale and Wolfe (1957) and Gould (1970) (see Dodson and Muller 1978). However, as pointed out by Little (1979), an alternative formulation that can be used to relate advertising to awareness (or sales) is the classic capital accumulation model suggested by Nerlove and Arrow (1962). Considering awareness as the form of goodwill, their model can be stated as:

$$dA/dt = f(u) - bA. \tag{13}$$

In equation (13), Nerlove and Arrow (1962) assume $f(u)$ to be a linear function. However, an extension of their model to a concave $f(u)$ function has been suggested by Gould (1970).

For equation (13), if we assume that $f(u)$ is an S-shaped function and that the impact of an advertising policy can be measured by total awareness, equation (7), then in Appendix C we show that Proposition 2 is valid for this model as well. That is, total awareness for the chattering policy is larger than that of a constant policy if and only if $f(u)$ is S-shaped (considering a concave function as the alternative, obviously). In addition, pulsing policy yields the same results as the chattering policy. That is, total awareness does not depend upon the number of pulses! Thus we conclude that even for the Nerlove-Arrow model, pulsing is optimal *only* under an S-shaped advertising response function.

Regarding the assumption of the S-shaped advertising response function, in addition to Sasieni (1970), the only other work, that we are aware of, that considers the optimality of a pulsing policy with an S-shaped function is the model proposed by Rao (1970). However, as concluded by Simon (1982, p. 353), the model proposed by Rao "neither provides any information on the timing or temporal sequence of the pulses nor substantiates a cyclic pulsation".

With respect to empirical studies examining the shape of the advertising response function, Simon (1971, Chapters 6–7) concludes that if there is any threshold level effect it must be present only in very small levels of advertising expenditure. Although, as documented by Little (1979) and Freeland and Weinberg (1980), some laboratory experiments have supported the hypothesis of the existence of the S-shaped response function. After reviewing several empirical studies on the shape of the advertising response function, Simon and Arndt (1980), however, conclude that advertising response functions with increasing returns (i.e. S-shaped function) have not been reliably observed in the laboratory or in the field. In fact, they report an overwhelming support for concave response functions—functions with diminishing returns. Our contention is that if firms follow a pulsing policy when faced with an S-shaped response function, the empirical studies based on aggregated advertising levels and responses *can not discover* an S-shaped function since the effect of pulsing is to linearize the convex part of the response function. Furthermore, allocation models suggesting concave approximation procedures in the presence of an S-shaped response function (e.g., Luss and Gupta 1975, Luss 1975, Weinberg 1977 and Freeland and Weinberg 1980) may yield inappropriate results. For example, Luss and Gupta (1975) quote Lodish (1971a) who in a different setting (CALLPLAN) proved that if one approximates an S-shaped response function by its concave envelope (i.e., in Figure 1 replace $f(u)$ by the straight line from the origin to $\bar{u}$ and then follow $f(u)$), then this ". . . yields a good approximation to the original model, provided . . . the available budget is sufficiently large".

In our setting, however, this can be misleading. If the total budget, B, is so large as to make $\alpha = 1$ (i.e. the firm can advertise at $\bar{u}$ in all the time periods) then the firm will do so, advertise at $\bar{u}$ for all t, and the shape of $f(u)$ up to $\bar{u}$ is irrelevant. As soon as B is such that the firm cannot advertise at $\bar{u}$ for all t (i.e. $\alpha < 1$), our analysis applies and the approximation can be misleading depending on the shape of $f(u)$. For example, suppose

we do approximate the function by its concave envelope but $f(u)$ is such that $y = f(\alpha \bar{u})$ = 0.02. Under this approximation, the optimal policy is blitz/maintenance policy. Concentrating on the maintenance part of the policy only, the real effect will be about *half* of the "approximated" effect which is by no means a good approximation. To see this observe that for Zielske's first group R_c for $y = 0.02$ is 7.4, while for a straight line the effect of advertising at $\alpha \bar{u}$ is $\alpha f(\bar{u}) = \alpha x$. Substituting αx into equation (11) (as in the proof of Proposition 3) one gets equation (10) for chattering. Thus the value of this "approximated" policy is the same as R_∞ which is 14.7.

Notice that the reason why we expect awareness to increase in the number of pulses k is that as k increases the firm is replicating more accurately the straight line of the concave envelope of $f(u)$. The exact replication can be achieved only with chattering.

A more recent work that discusses optimality of pulsing without an S-shaped function is the model suggested by Simon (1982).[8] Simon bases his model on empirical finding by Haley (1978) that an increase in advertising weights leads to a sharp immediate rise in sales. On the basis of this study he concludes that since ". . . contemporary models reproduce a monotonically increasing response of sales to an increase in advertising . . . we see that current models are structurally misspecified on an *a priori* basis." (p. 352)

Simon, therefore, specifies the following advertising response function:

$$f(u) = a + b \ln u + c \max \{0, \Delta u\} \tag{14}$$

for some parameters a, b, and c.

He then relates advertising to sales via a Nerlove-Arrow equation. Since the logarithmic function is concave, the new important feature of his model is the specification of the maximum function that depends upon Δu which is the change in advertising levels. This maximum function represents the *asymmetry* in the response. If Δu is positive, i.e., advertising increases, the advertising response becomes larger by $c\Delta u$. If, however, advertising decreases, the maximum function receives the value zero, and no change in the advertising response function is registered.

The model proposed by Simon (1982) is an important contribution to the literature and perhaps is the only model that yields pulsing as an optimal policy without an S-shaped response function. Some comments on this model, however, are warranted. First, the phenomenon observed by Haley can be explained by an S-shaped function. We do expect a rise in sales (or awareness) when advertising is increased. It is only the degree of change that is new in Haley's observation. This can (and likely is) explained by an S-shaped function where the lower level of advertising was in the convex (lower) part of $f(u)$ of Figure 1 while the higher level of advertising was in the concave part. The degree of curvature of the function (i.e., how close it is to be a step function) will determine the degree of change in the dependent variable (sales or awareness). Second, from a theoretical level it is clear that the asymmetry assumed by Simon directly implies that pulsing is optimal. If, by increasing the level of advertising, the firm gains and by reducing the level it does not lose (which is the exact effect of the maximum function in equation (14)) it is better for the firm to pulse as often as it can. In addition, no further investigation of the model is possible because of the appearance of $\ln u$ in equation (14). Thus when we set u to be zero to measure a pulsing policy, $\ln u$ is minus infinity.

The above discussion indicates that the model formulation suggested in this paper and its underlying assumptions are theoretically and empirically appealing.

Finally, it should be pointed out that although the focus of our exposition has been on the examination of the impact of alternative advertising policies on awareness, one

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[8] The possibility of pulsing without the S-shaped response function has also been explored by Näslund (1979) who utilizes the consumer behavior model proposed by Nicosia (1966). However, as discussed by Muller (1983), Näslund's analyses result into counter intuitive signs for some of the model parameters.

can transform our problem into profit maximizing objective by stating the following: the firm will maximize profits under a budget constraint of $B = \alpha \bar{u} T$ for $\alpha < 1$. Its revenues are linear in awareness, its cost function is a nonlinear function whose inverse is given by the S-shaped function $f(u)$ of Figure 1, and its dynamic equation is given by equation (2). There might be one pitfall in such an analysis. In case that there is a substantial difference between the decay rate for purchasing and the decay rate for advertising awareness, the form of schedule that maximizes total awareness may not be the optimal profit maximizing schedule.[9] We, however, hope that future empirical and theoretical extensions will attempt to examine the impact of advertising policies on sales and profits.[10]

[9] We are thankful to Hugh Zielske for bringing this point to our attention.
[10] This paper was received in September 1984 and has been with the authors for 2 revisions.

Appendix A

Substituting into R_k the value of $A(t)$ from equations (2) and (5), performing the integration and collecting terms yields:

$$R_k = A(T)/b + \alpha x T/(x + b) - \sum_{i=0}^{k-1} A((i + 1)T/k)/b + \sum_{i=0}^{k-1} A(iT/k)/(x + b) + x \sum_{i=0}^{k-1} A((i + \alpha)T/k)/b(x + b).$$

(A.1)

Advance the first summation by 1 (i.e., let $j = i + 1$) while noting end point conditions to get

$$\sum_{i=0}^{k-1} A((i + 1)T/k) = \sum_{j=1}^{k} A(jT/k) = A(T) + \sum_{i=0}^{k-1} A(iT/k).$$

Substituting this expression into R_k yields:

$$R_k = \alpha x T/(x + b) + x/b(x + b) \sum_{i=0}^{k-1} [A((i + \alpha)T/k) - A(iT/k)].$$

(A.2)

In order to compute $A(t)$, use equations (3) and (4) and the initial condition $A(0) = 0$ to arrive at:

$$A(t) = x(1 - e^{-(x+b)t})/(x + b) \quad \text{for} \quad 0 \le t \le \alpha T/k,$$

$$A(t) = x(e^{\alpha b T/k} - e^{-\alpha x T/k})e^{-bt}/(x + b) \quad \text{for} \quad \alpha T/k \le t \le T/k.$$

In the same way, the second cycle ($i = 1$) is computed using the above values for $A(T/k)$:

$$A(t) = x(e^{\alpha b T/k} - e^{-\alpha x T/k})e^{x T/k}e^{-(x+b)t}/(x + b)$$

$$+ x(1 - e^{(x+b)(T/k-t)})/(x + b) \quad \text{for} \quad T/k \le t \le (\alpha + 1)T/k,$$

$$A(t) = x(e^{\alpha b T/k} - e^{-\alpha x T/k})e^{-\alpha x T/k}e^{-bt}/(x + b) + x(e^{\alpha b T/k} - e^{-\alpha x T/k})e^{b T/k}e^{-bt}/(x + b)$$

$$\text{for} \quad (\alpha + 1)T/k \le t \le 2T/k.$$

In much the same way the ith cycle for $1 \le i \le k$ can be computed to be:

$$A(t) = Fe^{(i-1)bT/k}e^{ixT/k}e^{-(x+b)t} \sum_{j=0}^{i-1} e^{-j(\alpha x + b)T/k} + x(1 - e^{(x+b)(iT/k-t)})/(x + b)$$

(A.3)

for $iT/k \le t \le (i + \alpha)T/k$ for $1 \le i \le k - 1$, while for $i = 0$, $A(t)$ is given by the first term only

$$A(t) = Fe^{ibT/k}e^{-bt} \sum_{j=0}^{i} e^{-j(\alpha x + b)T/k} \quad \text{for} \quad (i + \alpha)T/k \le t \le (i + 1)T/k \text{ for } 0 \le i \le k - 1,$$

(A.4)

where F is given by:

$$F = x(e^{\alpha b T/k} - e^{-\alpha x T/k})/(x + b).$$

Computing (A.3) and (A.4) at iT/k and $(i + \alpha)T/k$ yields

$$A(iT/k) = Fe^{-bT/k} \sum_{j=0}^{i-1} e^{-j(\alpha x + b)T/k} \quad \text{for} \quad i \ge 1 \quad \text{and} \quad A(0) = 0,$$

(A.5)

$$A((i + \alpha)T/k) = Fe^{-\alpha bT/k} \sum_{j=0}^{i} e^{-j(\alpha x+b)T/k} \qquad \text{for} \qquad i \geq 0. \tag{A.6}$$

Let

$$B_i = \sum_{j=0}^{i-1} e^{-j(\alpha x+b)T/k},$$

$$B_i = 1 + e^{-(\alpha x+b)T/k} + e^{-2(\alpha x+b)T/k} + \cdots + e^{-(i-1)(\alpha x+b)T/k} = 1 + e^{-(\alpha x+b)T/k}[1 + e^{-(\alpha x+b)T/k} + \cdots + e^{-(i-2)(\alpha x+b)T/k}].$$

Add and subtract $e^{-(i-1)(\alpha x+b)T/k}$ to the square brackets to get:

$$B_i = 1 + e^{-(\alpha x+b)T/k}[B_i - e^{-(i-1)(\alpha x+b)T/k}].$$

Rearranging terms yields:

$$B_i = (1 - e^{-i(\alpha x+b)T/k})/(1 - e^{-(\alpha x+b)T/k}). \tag{A.7}$$

Substituting (A.5 and (A.6) into (A.2), performing the summation using (A.7) yields:

$$R_k = \alpha xT/(x + b) + x^2(1 - e^{-\alpha(x+b)T/k})L_k/b(x + b)^2, \tag{A.8}$$

where L_k is given by

$$L_k = B_k + (1 - e^{-(1-\alpha)bT/k})(k - B_k)/(1 - e^{-(\alpha x+b)T/k}). \tag{A.9}$$

(a) For a blitz policy, substitute $k = 1$ into L_k and R_k to yield equation (8).
(b) Collect terms of (A.8) and (A.9) to yield equation (9).
(c) A chattering policy is one at which there is an infinite number of pulses, i.e, $k \to \infty$. R_∞ can be computed as the limit of R_k when $k \to \infty$. The result is as follows:

$$R_\infty = \alpha xT/(x + b) + x^2 L/b(x + b)^2 \tag{A.10}$$

where L is given by

$$L = \alpha(x + b)[(1 - \alpha)bT + (1 - e^{-(\alpha x+b)T})\alpha(x + b)/(\alpha x + b)]/(\alpha x + b). \tag{A.11}$$

Collecting terms yields equation (10).

Appendix B

Let $z = \alpha(x + b)T$ and $w = (\alpha x + b)T$. Note that since $\alpha < 1$, it follows that $z < w$. Using equations (A.10), (A.11) and (8) yields:

$$R_\infty - R_1 = x^2/b(x + b)^2 w^2\{(1 - e^{-w})z^2 + (w - z)wz - (1 - e^{-z})w^2\}.$$

Clearly $R_\infty - R_1$ is positive if

$$(w + e^{-w} - 1)/w^2 \leq (z + e^{-z} - 1)/z^2. \tag{A.12}$$

(Note that $w + e^{-w} - 1 > 0$ since at $w = 0$ it is equal to zero and its derivative is positive.)

$$d[(w + e^{-w} - 1)/w^2]dw = (2 - w - we^{-w} - 2e^{-w})/w^3. \tag{A.13}$$

The R.H.s of equation (A.13) is negative since at $w = 0$ it is equal to zero and its derivative is proportional to $-1 + e^{-w} + we^{-w}$ which is negative (since at $w = 0$ it is zero and its derivative is negative). We conclude that (A.12) holds since $w > z$ and the derivative of the respective terms is negative.

Appendix C

The problem is to maximize R_k of equation (5) subject to the Nerlove-Arrow capital accumulation equation

$$dA/dt = f(u) - bA. \tag{A.14}$$

Using the same methods, mutatis mutandis, that led to equation (A.1) one arrives at the following:

$$R_k = A(T)/b + \alpha xT/b + (1/b) \sum_{i=0}^{k-1} [A(iT/k) - A((i + \alpha)T/k)].$$

Performing the summation, rearranging terms yields: $R_k = \alpha xT/b$. Therefore R_k is independent of k in this case. R_c for a constant policy of $\alpha \bar{u}$ is given by the following: $R_c = yT/b$. Thus $R_k > R_c$ if and only if $y < \alpha x$ which duplicates Proposition 2.

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