# **Tax Evasion and Financial Equilibrium**

Yoram Landskroner, Eitan Muller\*, and Itzhak Swary\*

# **I. Introduction**

Income tax evasion, the deliberate unreporting or underreporting of income, is a phenomenon of increasing economic importance and public concern. Despite empirical problems in measurement, recent estimates suggest that the problem is quite significant.<sup>1</sup>

This article extends the literature on tax evasion by presenting a new approach in the modeling of the decision to evade income taxes. The main innovation and contribution of the article is to introduce the option of investing in a risky asset as an alternative to tax evasion. The analysis of tax evasion is integrated with a model of financial market equilibrium.

A theoretical analysis of tax evasion decisions by individuals was first carried out by Allingham and Sandmo (1972) and Srinivasan (1973). Their models considered a risk-averse individual taxpayer who maximizes an expected utility function where the tax declaration decision is his only decision variable. The utility function has income as its single argument. Uncertainty is introduced into the model, via the probability that the taxpayer will be investigated by the tax authorities, who will then discover his true income and impose some penalty on the undeclared portion.

Allingham and Sandmo did not obtain clear results as to the impact of changes in actual income and tax rates on tax evasion. Unambiguous results were derived for the penalty rate and the probability of detection; an increase in either of these parameters will presumably induce an increase in declared income. Yitzhaki (1974) and more recently Koskela (1983) amended this analysis by considering an alternative form of the penalty function, in which the penalty is imposed on the evaded tax rather than on the evaded income. Yitzhaki concluded that, if taxpayers have decreasing absolute risk aversion, an increased tax rate will reduce the amount of evaded income; and declared income changes more slowly than true income. Poterba (1987), using time series data, finds that a 1% increase in the tax rate reduces compliance by 0.5-1%. These findings are consistent with findings of other studies, for example, Clotfelter (1983).

<sup>\*</sup> Faculty of Management, Tel Aviv University.

Address reprint requests to Yoram Landskroner, Jerusalem School of Business Administration, The Hebrew University of Jerusalem, Mt. Scopus, Jerusalem 91905.

<sup>&</sup>lt;sup>1</sup>The IRS (1988) estimates the gross income tax gap for tax year 1987 to be \$84.9 billion. This is the amount of income tax owed for the year but not voluntarily paid. The estimate does not include income from illegal transactions. Guttman (1977) estimated that, in 1976, the "subterranean economy" generated a GNP of \$176 billion, about 10% of the total "above ground" GNP.

Allingham and Sandmo (1972) recognized the incomplete nature of their analysis, in that the tax evasion decision was considered in isolation from other decisions that affect individuals' wealth. They have suggested two basic lines of research that could generalize their findings: (a) extension of the model, which would take the labor supply decision into account; and (b) extension that would incorporate portfolio decisions.<sup>2</sup> The first of these approaches has been undertaken in a number of studies, including those of Weiss (1976) and Sandmo (1981). In this article we have chosen the extension of the model that takes into account interrelatedness with the overall portfolio decision. The extension has been made in a framework of financial market equilibrium. Previous studies had concentrated on individual behavior, and had regarded tax evasion as an isolated decision; such studies did not derive financial equilibrium results. We were able to derive specific results by making some restrictive assumptions concerning individual preferences. We point out possible effects of these assumptions in some instances.

In conventional tax evasion models augmented by labor supply considerations, the individual's declared income and the number of work hours are endogenous variables affected by the tax structure. In such models the demand for leisure is actually the demand for a risk-free *asset*, where the (marginal) return is the (marginal) utility from leisure. On the other hand, increasing one's hours of work is regarded as risk taking. Accordingly, changes in the parameters of the tax structure would affect the hours of work. In the portfolio decision model used in this paper, an optimal position, which is affected by the tax structure, could be achieved by changing the individual's portfolio composition, for example, his position in risk-free and risky assets. In this context, if labor markets and financial markets are in equilibrium, it can be proved that the labor supply decision is separable from declared income and the composition of the individual's portfolio.<sup>3</sup>

In Section II we present a model in which the individual taxpayer faces two decisions simultaneously: the optimal choice of financial assets and the tax declaration decision. The investor faces two sources of uncertainty. The first is induced by his tax evasion decision, as in previous studies, and the second is introduced by a risky financial asset. We derive the individual optimal decision as well as the equilibrium conditions in the financial market. The effects of changes in the legal tax rate on individual decisions are considered in Section III. In Section IV we derive the effects of changes in true income on declared income. Most of Section V is devoted to the risk substitution property exhibited in our results. This property stems from the fact that we consider two types of risky assets, described earlier. Some concluding remarks are presented in the last section.

#### II. The Model

In our analysis we shall consider an economy in which two types of assets exist: financial assets representing claims on physical capital, and human capital. We distinguish between a risky and a riskless financial asset. The investors in our model are

<sup>&</sup>lt;sup>2</sup>Another possible extension is the analysis of tax evasion in the framework of optimal taxation theory.

<sup>&</sup>lt;sup>3</sup>For a discussion of the relations between labor market and financial market equilibrium, see Mayers (1974).

single-period maximizers of expected utility wealth. The investor's utility function conforms to the von Neumann Morgenstern axioms and exhibits risk aversion.

The investor's optimal decisions concern his investment in financial assets, and a tax declaration that is considered to be an additional type of risky investment. We assume differential taxation of human capital income and financial income, where the first is taxed at a higher rate than the second. This assumption may be justified by a tax structure that taxes wage income at higher rates than capital gains, such as the one that existed until recently in the United States. To simplify the algebra we assume that human capital is taxed at some constant positive rate, while financial income is not taxed (zero rate). However, our results, specifically those concerning the tax-induced substitutability between the two risky investments, are robust and hold also for positive but different tax rates. The analytical framework for the more general case is provided in Appendix 1.<sup>4</sup>

The formal choice situation of the tax paying investor is as follows: his budget constraint, determined by his initial wealth can be written as:

$$W = S_1 + S_0 + H, (1)$$

where  $S_1$  is the value of investment in the risky financial asset;  $S_0$  is investment in the riskless financial asset; and H is investment in human capital. The investor's total return (the wealth difference) is given by

$$R = S_1 r_1 + S_0 r_0 + H w (1 - t),$$
<sup>(2)</sup>

where  $r_1$ ,  $r_0$ , and w denote the rates of return on the three types of assets, respectively. By assumption, the return on human capital is certain; in contrast, the actual (effective) tax rate t is uncertain. This actual tax rate has a binomial distribution:

$$t = \begin{cases} \epsilon \tau \text{ with probability } (1 - p); \\ \tau(\lambda(1 - \epsilon) + \epsilon) \text{ with probability } p, \end{cases}$$
(3)

where  $\tau$  is the constant legal tax rate levied on human capital income;  $\epsilon$  is the decision variable of the investor, defined as the percentage of actual income declared;  $\lambda$  is a penalty rate imposed on the undeclared tax, which is greater than unity; p is some probability of the investor being audited by the tax authorities, who then find out his true income.<sup>5</sup> Equation (3) therefore formally presents the tax declaration decision: An investor declaring less than his actual income "invests" in a risky asset with some return and risk characteristics.

<sup>&</sup>lt;sup>4</sup>It should be noted that the largest source of the tax gap is associated with human capital income in the form of informal suppliers' and other nonfarm proprietors' income (including roadside or sidewalk vendors, moonlighting craftsmen or mechanics, unlicensed providers of child or elderly care services, and similar operators). In 1987 these sources constituted about 29% of the total tax gap and about 38% of individual income tax gap (IRS, 1988).

<sup>&</sup>lt;sup>5</sup>Alexander and Feinstein's (1986) econometric study of tax evasion deals with, among other things, the fact that IRS auditors don't always succeed in detecting evasion.

To derive explicit optimal decision rules and financial equilibrium results, a meanvariance framework is used. Thus, the investor will maximize a mean-variance utility function of his wealth difference or income.<sup>6</sup> The expected value of the total return can be expressed as follows:

$$E(R) = S_1 E(r_1) + S_0 r_0 + Hw(1 - E(t)), \qquad (4)$$

where E denotes the expected value operator. The expected effective tax rate E(t) can be derived by using (3) as:

$$E(t) = \epsilon(\tau - \tau\lambda p) + p\tau\lambda = \epsilon\tau(1 - p\lambda) + p\tau\lambda.$$
(5)

The expected tax rate should increase with declared income; we therefore require that  $p\lambda < 1$ . This also assures that E(t) < 1 since, when  $\epsilon = 1$ ,  $E(t) = \tau < 1$ . Similarly, the variance of the wealth difference is given by

$$var(R) = S_1^2 var(r_1) + H^2 w^2 var(t) - 2S_1 H w cov(r_1, t).$$
(6)

There is no a priori reason to believe that there is a dependence between the rate of return on the risky financial asset and the effective tax rate; at least there does not seem to be a good reason to assume that the statutory tax rate  $\tau$ , and the return on the risky financial asset are correlated. Therefore it is assumed that  $cov(r_1, t) = 0$ , which also makes the problem more tractable.

The variance of the effective tax rate is

$$\operatorname{var}(t) = p(1-p)\tau^2\lambda^2(1-\epsilon)^2. \tag{7}$$

As is well known, the mean-variance criterion is equivalent to expected utility maximization for the quadratic utility function. This is obviously a restrictive assumption, but this is a cost one has to pay in order to be able to derive explicit equilibrium results. In a recent paper Meyer (1987) identifies a condition sufficient to ensure consistency of two-moment models with expected utility maximization and confirms that it holds in many economic models.

We adopt this assumption so that the utility function is specified to be

$$U(R) = \alpha R - \beta R^2, \qquad (8)$$

where we restrict the function to  $\beta > 0$  and  $\alpha > 2\beta R$ , to be consistent with the assumption of risk aversion. To find the optimal decision, the investor maximizes the expected utility of equation (8) with respect to the two decision variables: investment in the risky financial asset  $S_1$ , and the tax declaration  $\epsilon$ . By assumption the optimal investment in human capital has already been made. The maximization of (8) is subject to the budget constraint of equation (1). Thus, we obtain the two first-order conditions

<sup>&</sup>lt;sup>6</sup>Assuming a specific utility function (thus also specific risk aversion) allows us to derive explicit equilibrium results. An alternative approach would be a more general one that allows for different attitudes towards risk, but the results would be more general. One would not be able under the more general approach to obtain explicit equilibrium results such as the demand function for the risky asset.

for the investor. First for the tax declaration  $\epsilon$  it is

$$\frac{1}{2\beta} \cdot \frac{\partial EU}{\partial \epsilon} = -\left(\frac{\alpha}{2\beta} - E(R)\right) (Hw\tau(1-p\lambda)) + H^2 w^2 p(1-p)\tau^2 \lambda^2 (1-\epsilon)$$
$$= 0. \tag{9}$$

Similarly, for the optimal investment in the risky financial asset,  $S_1$ , we obtain:

$$\frac{1}{2\beta}\frac{\partial E U}{\partial S} = -\left(\frac{\alpha}{2\beta} - E(R)\right)E(r_1 - r_0) - S_1 \operatorname{var}(r_1) = 0.$$
(10)

The two first-order conditions are similar, in that they consist of two types of terms. First is a term that reflects attitudes toward risk of the investor, as expressed in the utility function; this term is  $\alpha/2\beta - E(R)$ ; second are terms reflecting the risk and return characteristics of the investment:  $E(r_1)$ ,  $r_0$ , and  $var(r_1)$  for the financial assets, and similar terms for human capital and the tax declaration. These first-order conditions can be interpreted as equating the subjective marginal rate of substitution between risk and return with the objective marginal rate of transformation.

To obtain financial equilibrium results, we sum equation (10) over all investors, obtaining

$$\gamma = \frac{E(r_1 - r_0)}{S_1 \operatorname{var}(r_1)},$$
(11)

where  $S_1 = \Sigma S_1$  is the summation over investors and also

$$\gamma = \left[\Sigma \frac{\alpha}{2\beta} - E(R)\right]^{-1},\tag{12}$$

where  $\gamma$  is a measure of absolute risk aversion in the market, similar to the Arrow-Pratt measure of risk aversion. As seen in (12), this measure is a harmonic mean of individual measures of risk aversion. According to the equilibrium condition of (11), this market measure of risk aversion determines the market risk premium per unit of risk, which is the RHS of (11). Results similar to (10) and (11) are well known in finance literature. Finally, we may rewrite (11) to obtain the market demand for the risky financial asset:

$$S_{1} = \Psi/\gamma$$

$$= \Psi \Sigma \frac{\alpha}{2\beta} - \Psi \Sigma E(R)$$

$$= \Psi \sigma - \Psi \{ E(r_{1} - r_{0}) S_{1} + r_{0}(W - H) + w \Sigma H (1 - p\tau\lambda - \epsilon\tau(1 - p\lambda)) \}$$
(13)

where

$$\Psi = \frac{E(r_1 - r_0)}{\operatorname{var}(r_1)}$$

This is known in the literature as the market price of risk, and where  $\sigma = \sum \alpha / 2\beta$ , and  $W = \sum W$ ,  $H = \sum H$ .

### **III. Tax Effects**

One of the main issues to be considered in this article is the effect of changes in the legal tax rate on the total amount of evaded income.

In the Allingham-Sandmo study, the investor paid the penalty  $\lambda$  on undeclared income; therefore, the effective tax rate had the following binomial distribution:

$$t = \begin{cases} \epsilon \tau + \lambda (1 - \epsilon) \text{ with probability } p; \\ \epsilon \tau \text{ with probability } (1 - p), \end{cases}$$
(14)

where  $\epsilon$  was the percentage of actual income declared. They concluded that there are two opposing effects (of income and substitution) when tax rate  $\tau$  is changed, and thus no clear answer was found to satisfy the above question.

In our setting, the penalty is imposed on the undeclared tax, as in equation (3); more importantly, the investor can invest in risky asset  $S_1$ . To clearly see the effect of the inclusion of a risky financial asset, define  $h = H/\Sigma H$  and  $e = \Sigma h\epsilon$ . Thus, e is the average percentage of declared income where the weights of this average are each investor's share of investment in human capital.

From equation (13), the market demand for the risky financial asset is

$$S_{1} = \frac{\Psi}{1 + \Psi E(r_{1} - r_{0})} \{ \sigma - r_{0}(W - H) - wH(1 - p\tau\lambda) + wH\tau(1 - p\lambda)e \}.$$
(15)

In the same manner, one can sum equation (9) over investors, substitute from (13) for the term  $\sigma - \Sigma E(R)$ , and rearrange terms to arrive at the solution for the average percentage of undeclared income (1 - e):

$$1 - e = S_1(1 - p\lambda)/\Psi wHp(1 - p)\tau\lambda^2.$$
(16)

Equations (15) and (16) form a set of two equations, with the two unknowns e and  $S_1$ . We first wish to find out the conditions for an interior solution.

From equation (10), it is clear that  $S_1 > 0$ , because of the assumptions made on  $\alpha$  and  $\beta$ . From equation (5),  $p\lambda < 1$ , and therefore  $\epsilon < 1$ . One can check that  $\partial^2 EU/\partial \epsilon^2 < 0$ ; thus, to assure the positivity of e, we must require that  $\partial EU/\partial \epsilon > 0$  at  $\epsilon = 0$ . We assume that these inequalities hold, and are thereby guaranteed an interior solution. One can now solve equations (15) and (16) to arrive at the following equations for  $S_1$  and e:

$$S_{1} = \Psi \frac{\sigma - r_{0}(W - H) - wH(1 - \tau)}{1 + \Psi E(r_{1} - r_{0}) + (1 - p\lambda)^{2}/p(1 - p)\lambda^{2}},$$
(17)

$$1 - e = \frac{(1 - p\lambda) \left[ \sigma - r_0 (W - H) - w H (1 - \tau) \right]}{w H p (1 - p) \tau \lambda^2 \left[ 1 + \Psi E (r_1 + r_0) + (1 - p\lambda)^2 / p (1 - p) \lambda^2 \right]}.$$
 (18)

We have shown that  $S_1$  is positive for all values of  $\tau$ . It is reasonable to assume that this will hold in the limit as well as when  $\tau$  is zero; in other words, even when income

is not taxed, the demand for the risky asset will be positive. Under this assumption, it is cumbersome but rather straightforward to show that  $\partial e/\partial \tau > 0$ . Note from (17) that  $\partial S_1/\partial \tau > 0$ . Thus, when the tax rate is increased, it causes the penalty associated with  $\epsilon$  to increase, thus increasing the risk associated with evasion. The investor then finds  $S_1$  a more attractive investment.

It is of interest to compare the results with that obtained by Yitzhaki (1974), whose setting was similar to ours in that the penalty was imposed on the evaded tax. He found that, if taxpayers have absolute risk aversion that *decreases* with income, a tax increase would increase the proportion of income declared. This result is due to the fact that no substitution effect occurs in his setting and the income effect causes  $\partial e/\partial \tau$  to be positive. The same result is obtained here, but the driving force is the ability to take the risk in the form of investment in the risky asset  $S_1$ .

# **IV. Income Effects**

In the previous section we discussed the effects of tax rate changes on tax evasion; in this section we address the problem of the effects of changes in true income on declared income. Yitzhaki (1974) concluded that declared income changes more slowly than true income. Pencavel (1979), however, questioned the validity of this result, on the grounds that Yitzhaki's analysis was limited because it did not include the decision of the individual to allocate his or her time between work and leisure.<sup>7</sup>

The effect of a change in true taxable income on declared income is obtained by differentiating e with respect to the wage rate w using equation (18). It is a straightforward matter to confirm that  $\partial e/\partial w > 0$ . Note that, since H is constant, a change in w is equivalent to a change in true taxable income. We conclude, therefore, that as true taxable income increases, the fraction declared increases. This result seems to confirm Pencavel's criticism; however, we have not included in our model the question of time allocation between work and leisure, as did Pencavel. On the other hand we have expanded the opportunity set faced by the investor, to include a risky financial asset as well as the risky tax declaration decision.

The result may be explained by the risk attitude of the individual investors in our model. As stated, according to our specifications the investor's utility function exhibits increasing risk aversion. Thus, as true income increases, other factors being the same, he will reduce his investment in risky assets (concommitantly with the size of the tax evasion as well as holdings in risky financial assets  $S_1$ ). The latter can be easily confirmed by differentiating  $S_1$  with respect to w, using equation (17). This result does not contradict that of Yitzhaki, because he assumed decreasing risk aversion.

# V. Substitutability Property

We have shown in Section III that when the tax rate is increased, investors will increase their holdings of the risky asset  $S_1$  and decrease their tax evasion. This is because a tax increase simulates a shift in investment to the tax-free investment  $S_1$ . Since it is a risky

<sup>&</sup>lt;sup>7</sup>Pencavel (1979) claimed that Yitzhaki's result was that the fraction of declared income falls with increases in true income. Yitzhaki, however, stated that declared income changes more slowly than true income. The latter is a weaker statement, which does not seem to imply the first assertion.

investment, it causes the investors to be more risk-averse in tax declaration and therefore to evade less in taxes. Thus, investors substitute the risk inherent in tax evasion for the risk of investment in  $S_1$ .

This substitutability property is apparent with respect to other parameters of the model as well. A higher risk or lower payoff in one asset, for example, causes a shift to the other asset. An increase in either p or  $\lambda$  will simultaneously cause an increase in both  $S_1$  and  $\epsilon$ . Thus, if either the penalty or the probability of an audit is increased, there will be a shift from tax evasion to other risky assets, that is, there will be reduction in the amount of tax evaded  $(1 - \epsilon)$  and an increase in the investment of  $S_1$  (see Appendix 2). As noted, this is consistent with earlier results (e.g., Allingham and Sandmo).

In this sense there is no difference between a tax increase, an increase in the penalty, or an increase in the probability of an audit. Their effect will be to increase the variance of the effective tax rate (in case of p, if p < 1/2) and simultaneously to increase the expected tax rate (in case of  $\lambda$ , if  $p > \epsilon$ ; see equations (5) and (7)). The investor will reduce the variance by decreasing the amount of tax evaded and increase holdings in the relatively more attractive investment of  $S_1$ .

Analogously, an increase in the variance of the investment  $S_1$ ,  $var(r_1)$ , will cause a decrease in  $\Psi$ , and thus a decrease in the amount invested in  $S_1$ , and an increase in the amount of tax evaded.<sup>8</sup>

#### **VI.** Concluding Remarks

We have examined the tax evasion decision by an individual in a context of financial equilibrium. Tax evasion has been viewed as a demand for a risky asset and analyzed in a mean-variance framework. Two main issues in the literature were examined, the first of which is the effect of changes in the legal tax rate on evaded income. An increase in the tax rate will cause an increase in the declared income; unlike Allingham and Sandmo, we were able to derive a clear result. Our result is similar to that of Yitzhaki; the driving force behind his result, however, was that an individual's utility function exhibits decreasing risk aversion. Our result, on the other hand, was due to the fact that the investor can substitute an investment in a taxable asset for an investment in a nontaxable asset, and vice versa.

We also considered the effect of changes in true income on declared income. We found that, as true taxable income increases, the fraction declared increases, a result that differs from previous findings because of different assumptions about the risk characteristics of the utility function. A third important result concerns the substitution property between the two types of risky assets, financial versus human capital (including tax evasion). A higher risk or lower expected return in one asset causes a shift in investment to the other.

The basic framework for analyzing tax evasion proposed by Allingham and Sandmo

<sup>&</sup>lt;sup>8</sup>It may be asked whether it is possible to have a simultaneous increase both in the amount of investment  $S_1$  and in tax evasion. This query is answered affirmatively as follows: The Arrow-Pratt measure of risk aversion for each investor is defined by  $1/(\alpha/2\beta - R)$ . If some of the investors become more risk-loving, then  $\sigma = \sum \alpha/2\beta$  will increase. This will cause investors to increase their holdings of the risky asset and to simultaneously evade more taxes. Since  $\sigma$  is an aggregate measure, the above result will hold if some investors become more and some less risk-averse so that the sum  $\sum \alpha/2\beta$  becomes larger.

has been extended by several authors to include the labor supply decision. In this article we have incorporated tax evasion into the individual's portfolio decision but have disregarded labor supply. To a certain extent, these two extensions are interchangeable. There is a certain resemblance between the riskless financial asset in our model and leisure in models that consider labor supply. The similarity is that both present a riskless "investment" opportunity; on the other hand, these assets differ in that the return on leisure is unique to each investor according to her or his utility function, while the return on the riskless asset is the same for all investors.

#### Appendix 1

We present here the framework of the model for the general case, in which both financial income as well as human capital income are taxed but at different rates.

 $\tau$  and  $\tau_1$  ( $\tau > \tau_1 > 0$ ) are the statutory tax rates of human capital and financial income, respectively. The investor's total return is

$$R = (S_1 r_1 + S_0 r_0)(1 - t_1) + Hw(1 - t).$$
(2a)

The effective tax rate on human capital income is as before:

$$t = \begin{cases} \epsilon \tau \text{ with probability } (1-p); \\ \tau(\lambda(1-\epsilon) + \epsilon) \text{ with probability } p. \end{cases}$$
(3a)

The effective tax rate on financial income is

$$t_{1} = \begin{cases} \epsilon_{1}\tau_{1} \text{ with probability } (1 - p_{1}); \\ \tau_{1}(\lambda(1 - \epsilon_{1}) + \epsilon_{1}) \text{ with probability } p_{1}. \end{cases}$$
(3b)

Note that the two effective tax rates are different owing to different statutory rates but also to different probabilities of detection as well as different tax evasion proportions. The expected total return is

$$E(R) = S_1 E(r_1 \cdot (1 - t_1)) + S_0 r_0 (1 - E(t_1)) + Hw(1 - E(t))$$
  
=  $S_1 [(1 - E(t_1))E(r_1) - cov(r_1, t_1)]$   
+  $S_0 r_0 (1 - E(t_1)) + Hw(1 - E(t)).$  (4a)

The variance (var) of the total return is given by

$$\operatorname{var}(R) = S_{1}^{2} \left[ \operatorname{var}(r_{1}) + \operatorname{var}(r_{1}t_{1}) \right] + S_{0}^{2}r_{0}^{2}\operatorname{var}(t_{1}) + H^{2}w^{2}\operatorname{var}(t) - 2S_{1} \left\{ S_{1}\operatorname{cov}(r_{1}, r_{1}t_{1}) + S_{0}r_{0} \left[ \operatorname{cov}(r_{1}, t_{1}) - \operatorname{cov}(r_{1}t_{1}, t_{1}) \right] + Hw \left[ \operatorname{cov}(r_{1}, t) - \operatorname{cov}(r_{1}t_{1}, t) \right] \right\} + 2S_{0}r_{0}Hw \operatorname{cov}(t_{1}, t).$$
(6a)

These are obviously cumbersome equations. In order to obtain tractable results we assumed that  $\tau_1 = 0$ ; these are the results presented in the main body of the article.

# **Appendix 2**

The parametric analysis with respect to the parameters  $\lambda$  and p is performed as follows, by using equations (17) and (18).

It is rather straightforward to show the positivity of  $\partial S_1 / \partial \lambda$  and  $\partial S_1 / \partial p$ , and therefore it is omitted. From equation (18) we obtain

$$\partial e/\partial \lambda = pA[\lambda(1+\Psi E(r_1-r_0))(1-p)(2-p\lambda)-(1-p\lambda^2)], \qquad (A1)$$

where

$$A = \frac{wH\tau[\sigma - r_0(W - H) - wH(1 - \tau)]}{\left[wHp(1 - p)\tau\lambda^2(1 + \Psi E(r_1 - r_0)) + wH\tau(1 - p\lambda^2)\right]^2}.$$
 (A2)

Following equation (17), A is positive, because  $S_1$  is positive as well. Thus, we must show the positivity of the following expression:

$$\lambda(1+\Psi E(r_1-r_0))(1-p)(2-p\lambda)-(1-p\lambda)^2.$$

Because  $\Psi E(r_1 - r_0) > 0$ , the above expression is larger than  $F(\lambda)$ , where  $F(\lambda) = \lambda(1 - p)(2 - p\lambda) - (1 - p\lambda)^2$ .

It is straightforward to confirm that F(1) > 0 and  $dF/d\lambda > 0$ . Since  $\lambda > 1$ , it follows that  $F(\lambda) > 0$ .

From equation (18) it follows that

$$\frac{\partial e}{\partial p} = \lambda A \Big[ (1 - 2p + p^2 \lambda) \lambda (1 + \Psi E (r_1 - r_0)) - (1 - p \lambda)^2 \Big], \qquad (A3)$$

where A is defined in equation (A2). Since  $\Psi E(r_1 - r_0) > 0$ ,

$$\partial e/\partial p = \lambda A [(1-p^2\lambda)\lambda - (1-p\lambda)^2] = \lambda A(\lambda-1) > 0.$$

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#### References

Alexander, C., and Feinstein, J. May 1986. A Microeconomic Analysis of Income Tax Evasion. Working Paper, MIT.

Allingham, M. G., and Sandmo, A. 1972. Income Tax Evasion: A Theoretical Analysis. Journal of Public Economics 1:323-338.

- Clotfelter, C. T. 1983. Tax Evasion and Tax Rates: An Analysis of Individual Returns. *Review* of Economics and Statistics 65:363-373.
- Guttman, P. M. November/December 1977. The Subterranean Economy. Financial Analysts Journal 33:26-27.

Internal Revenue Service. March 1988. Income Tax compliance Research: Gross Tax Gap Estimates and Projections for 1973-1992. Publication 7285. Washington, DC: IRS.

- Koskela, E. 1983. On the Shape of Tax Schedule, the Probability of Detection, and the Penalty Schemes as Deterrents to Tax Evasion. *Public Finance*. 38:70-80.
- Mayers, D. 1974. Portfolio Theory, Job Choice and the Equilibrium Structure of Expected Wages. Journal of Financial Economics 1:23-42.
- Meyer, J. 1987. Two-Moment Decision Models and Expected Utility Maximization. American Economic Review 77:421-430.
- Pencavel, J. H. 1979. A Note on Income Tax Evasion, Labor Supply and Non-Linear Tax Schedules. *Journal of Public Economics* 12:115-124.
- Poterba, M., January 1987. Tax Evasion and Capital Gains Taxation. Working Paper, MIT and NBER.
- Sandmo, A. 1981. Income Tax Evasion, Labor Supply, and the Equity-Efficiency Tradeoff. Journal of Public Economics 16:265-288.

Srinivasan, T. N. 1973. Tax Evasion: A Model. Journal of Public Economics 2:339-346.

- Weiss, L. 1976. The Desirability of Cheating Incentives and Randomness in the Optimal Income Tax. Journal of Political Economy 84:1343-1352.
- Yitzhaki, S. 1974. A Note on Income Tax Evasion: Theoretical Analysis. Journal of Public Economics 3:201-202.